Your name:

Instructor (please circle): Alex Barnett Naomi Tanabe

Math 22 Fall 2016, Homework 3, due Wed Oct 5

Please show your work. No credit is given for solutions without work or justification.

(1) Let T be a linear transformation given by $T(x_1, x_2) = \left(\frac{3}{2}x_1 + \frac{1}{2}x_2, x_1 - x_2, x_2\right)$.

(a) Find the standard matrix A for the transformation T:

(b) Find a vector $\boldsymbol{x}$ whose image under T is $(5, 2, 1)$:

(c) What is the definition for a linear transformation $T : \mathbb{R}^n \to \mathbb{R}^m$ to be one-to-one?

(d) Is this transformation T one-to-one? Justify your answer.

(2) Let $A = \begin{bmatrix} 0 & 1 & 0 \\ 1 & 2 & 0 \\ 2 & 4 & 1 \end{bmatrix}$

(a) If A is not invertible, prove it, else, if A is invertible, find A^{-1}:

(b) Use part (a) to express the vector $(4, 1, -1)$ as a linear combination of the columns $\mathbf{a}_1$, $\mathbf{a}_2$, $\mathbf{a}_3$ of A:

(c) Now let A be a general invertible $n \times n$ matrix. Prove that A^T is invertible. [Note: use only the definition of invertible and properties of the transpose. Make sure every object you use exists! Do not use the IMT $\ddot{\smile}$]

(3) An alien species has two forms: "larval" and "flying." Each year half the larval turn into flying ones, with the other half staying larval. Also, each year, 10% of the flying ones die, and the others stay flying.

(a) Stacking the larval and flying populations into a column vector $\mathbf{x}$, write the population vector at year $k+1$ in terms of that at year k. Be sure to give the matrix.

(b) If the populations at "year zero" (the year the aliens land on Earth...) are 200 larval and 0 flying, what are the populations at year 3?

BONUS: What is the long-term fate of the alien race, i.e. the vector $\lim_{k\to\infty} \mathbf{x}^{(k)}$?