MATH 22 WORKSHEET : Basis for Col A

7/26/06

Consider $A = \begin{bmatrix} 1 & 2 & 1 & 2 \\ 2 & 4 & 0 & 2 \\ 3 & 6 & 0 & 3 \end{bmatrix} = \begin{bmatrix} \vec{a}_1 & \vec{a}_2 & \vec{a}_3 & \vec{a}_4 \end{bmatrix}$

Are the columns linearly independent? If not, find any linear dependence relations between the columns.

e.g. $\vec{a}_3 + \vec{a}_1 = \vec{a}_2$

Do this by row reducing:

A in REF : $\begin{bmatrix} \cdot & \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot \end{bmatrix} = \begin{bmatrix} \vec{b}_1 & \vec{b}_2 & \vec{b}_3 & \vec{b}_4 \end{bmatrix}$

What are the dependence relations between cols $\vec{b}_1 \cdots \vec{b}_4$? Different from those of $\vec{a}_1 \cdots \vec{a}_4$? Why?

Look at the REF : why is $\vec{b}_2$ in Span$\{\vec{b}_1\}$?
why is $\vec{b}_4$ in Span$\{\vec{b}_1, \vec{b}_3\}$?
What is the general rule here? Which cols can be written in terms of which others?

Which columns of A can you choose to give a basis for Col A?

Write the general rule for finding a basis for Col A :

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~ SOLUTIONS ~

Consider $A = \begin{bmatrix} 1 & 2 & 1 & 2 \\ 2 & 4 & 0 & 2 \\ 3 & 6 & 0 & 3 \end{bmatrix} = \begin{bmatrix} \vec{a}_1 & \vec{a}_2 & \vec{a}_3 & \vec{a}_4 \end{bmatrix}$

Are the columns linearly independent? If not, find any linear dependence relations between the columns.

↳ no, since there are 4 vectors in $\mathbb{R}^3$.

e.g. $\vec{a}_3 + \vec{a}_1 = \vec{a}_2$

Do this by row reducing:

$$A \sim \begin{bmatrix} 1 & 2 & 1 & 2 \\ 0 & 0 & -2 & -2 \\ 0 & 0 & -3 & -3 \end{bmatrix} \sim \begin{bmatrix} 1 & 2 & 1 & 2 \\ 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

Reading from REF:

$$x_1 = -2x_2 - x_4$$
$$x_2 = x_2$$
$$x_3 = -x_4$$
$$x_4 = x_4$$
$$= \begin{bmatrix} -2 \\ 1 \\ 0 \\ 0 \end{bmatrix} s + \begin{bmatrix} -1 \\ 0 \\ -1 \\ 1 \end{bmatrix} t$$

x_2 free x_4 free

A in REF: $\begin{bmatrix} 1 & 2 & 0 & 1 \\ 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 0 \end{bmatrix} = \begin{bmatrix} \vec{b}_1 & \vec{b}_2 & \vec{b}_3 & \vec{b}_4 \end{bmatrix}$

What are the dependence relations between cols $\vec{b}_1 \cdots \vec{b}_4$? Different from those of $\vec{a}_1 \cdots \vec{a}_4$? Why?

Dependence relations include: $-2\vec{a}_1 + \vec{a}_2 = \vec{0}$, $-\vec{a}_1 - \vec{a}_3 + \vec{a}_4 = \vec{0}$

No, column dependences are the same, because row operations preserve them.

Look at the REF: why is $\vec{b}_2$ in Span$\{\vec{b}_1\}$? only 1st entry.

why is $\vec{b}_4$ in Span$\{\vec{b}_1, \vec{b}_3\}$? only 1st & 2nd entries, covered by cols 1, 3.

What is the general rule here? Which cols can be written in terms of which others?

Any column can by written as lin. comb. of pivot columns up to that point.

Which columns of A can you choose to give a basis for Col A?

Just the pivot columns, since column relations in $\vec{a}_1 \cdots \vec{a}_4$ identical to those in $\vec{b}_1 \cdots \vec{b}_4$.

Write the general rule for finding a basis for Col A:

Find pivot columns, take just those original columns of A.