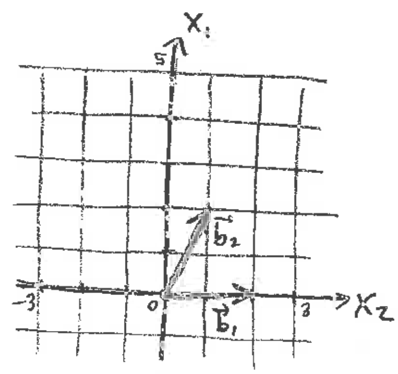
# Basis representation WORKSHEET (basics only)

10/27/03
Alex Barnett

Basis for $\mathbb{R}^2$ from lecture: $B = \{\vec{b}_1, \vec{b}_2\}$, with $\vec{b}_1 = \begin{bmatrix} 2 \\ 0 \end{bmatrix}$, $\vec{b}_2 = \begin{bmatrix} 1 \\ 2 \end{bmatrix}$

A) If $[\vec{x}]_B = \begin{bmatrix} -1 \\ 2 \end{bmatrix}$, what is $\vec{x}$ ?

Plot $\vec{x}$ and show how it is built from a lin. comb. of $\vec{b}_1, \vec{b}_2$ :

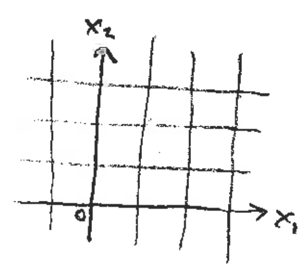



B) If $\vec{x} = \begin{bmatrix} 3 \\ 3 \end{bmatrix}$, what is $[\vec{x}]_B$ ?

Hint: write $[\vec{x}]_B = \begin{bmatrix} c_1 \\ c_2 \end{bmatrix}$, then $c_1 \vec{b}_1 + c_2 \vec{b}_2 = \vec{x}$

Can you solve for $c_1, c_2$ ? Is it unique? Why?

Show how this $\vec{x}$ is built from a linear comb. of $\vec{b}_1, \vec{b}_2$



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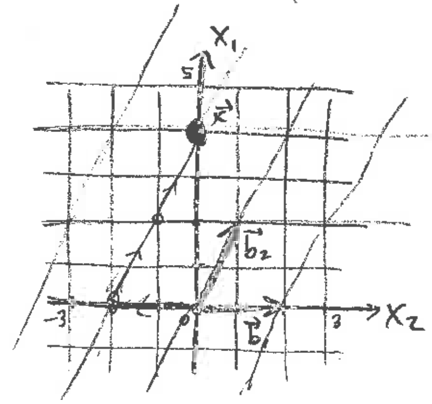
A) If $[\vec{x}]_B = \begin{bmatrix} -1 \\ 2 \end{bmatrix}$, what is $\vec{x}$?

$\vec{x} = -1\begin{bmatrix} 2 \\ 0 \end{bmatrix} + 2\begin{bmatrix} 1 \\ 2 \end{bmatrix}$

Plot $\vec{x}$ and show how it is built from a lin. comb. of $\vec{b}_1, \vec{b}_2$:




$P_B = \begin{bmatrix} 2 & 1 \\ 0 & 2 \end{bmatrix}$

B) If $\vec{x} = \begin{bmatrix} 3 \\ 3 \end{bmatrix}$, what is $[\vec{x}]_B$? solve $P_B \vec{c} = \vec{x}$

Hint: write $[\vec{x}]_B = \begin{bmatrix} c_1 \\ c_2 \end{bmatrix}$, then $c_1\vec{b}_1 + c_2\vec{b}_2 = \vec{x}$

Can you solve for $c_1, c_2$? Is it unique? Why?

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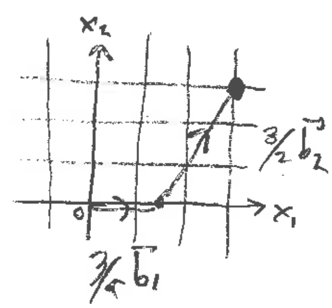
$$\left[\begin{array}{cc|c} 2 & 1 & 3 \\ 0 & 2 & 3 \end{array}\right] \sim \left[\begin{array}{cc|c} 1 & 1/2 & 3/2 \\ 0 & 1 & 3/2 \end{array}\right] \sim \left[\begin{array}{cc|c} 1 & 0 & 3/4 \\ 0 & 1 & 3/2 \end{array}\right]$$

Show how this $\vec{x}$ is built from a linear comb. of $\vec{b}_1, \vec{b}_2$




or, invert: $P_B^{-1} = \frac{1}{2\cdot 2 - 0}\begin{bmatrix} 2 & -1 \\ 0 & 2 \end{bmatrix}$

$= \begin{bmatrix} 1/2 & -1/4 \\ 0 & 1/2 \end{bmatrix}$

Use $[\vec{x}]_B = P_B^{-1}\vec{x} = \begin{bmatrix} 1/2 & -1/4 \\ 0 & 1/2 \end{bmatrix}\begin{bmatrix} 3 \\ 3 \end{bmatrix} = \begin{bmatrix} 3/4 \\ 3/2 \end{bmatrix}$.

See book for more theoretical/abstract questions. (this was just for $\mathbb{R}^n$)