MATH 22 WORKSHEET : Matrix Inverse

Barrett
7/10/06

a) Find A^{-1} for $A = \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}$

first find
$ad - bc = -2$

Is it zero? What does this immediately say about invertibility of A?

ans: $A^{-1} = \begin{bmatrix} \quad \end{bmatrix}$

b) Use above A^{-1} to solve $A\vec{x} = \vec{b}$
for $\vec{b} = \begin{bmatrix} 1 \\ -1 \end{bmatrix}$ <u>without</u> row reduction:

now check that $A\vec{x}$ is indeed equal to $\vec{b}$!

$\vec{x} = ? \qquad = \begin{bmatrix} \quad \end{bmatrix}$

check answer
Exercise 2.2.31

c) Row reduce $[A \mid I]$ to compute A^{-1} for $A = \begin{bmatrix} 1 & 0 & -2 \\ -3 & 1 & 4 \\ 2 & -3 & 4 \end{bmatrix}$:

$$\begin{bmatrix} 1 & 0 & -2 & 1 & 0 & 0 \\ -3 & 1 & 4 & 0 & 1 & 0 \\ 2 & -3 & 4 & 0 & 0 & 1 \end{bmatrix} \sim \begin{bmatrix} \quad \end{bmatrix} \sim$$

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SOLUTION

a) Find A^{-1} for $A = \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}$

first find $ad - bc = -2$

Is it zero? no. What does this immediately say about invertibility of A? It's invertible!

ans: $A^{-1} = \frac{1}{-2}\begin{bmatrix} 4 & -2 \\ -3 & 1 \end{bmatrix} = \begin{bmatrix} -2 & 1 \\ 3/2 & -1/2 \end{bmatrix}$

b) Use above A^{-1} to solve $A\vec{x} = \vec{b}$ for $\vec{b} = \begin{bmatrix} 1 \\ -1 \end{bmatrix}$ without row reduction:

$$\vec{x} = A^{-1}\vec{b} = \begin{bmatrix} -2 & 1 \\ 3/2 & -1/2 \end{bmatrix}\begin{bmatrix} 1 \\ -1 \end{bmatrix} = \begin{bmatrix} -3 \\ 2 \end{bmatrix}$$

check answer Exercise 2.2.31

c) Row reduce $[A \mid I]$ to compute A^{-1} for $A = \begin{bmatrix} 1 & 0 & -2 \\ -3 & 1 & 4 \\ 2 & -3 & 4 \end{bmatrix}$:

$$\left[\begin{array}{ccc|ccc} 1 & 0 & -2 & 1 & 0 & 0 \\ -3 & 1 & 4 & 0 & 1 & 0 \\ 2 & -3 & 4 & 0 & 0 & 1 \end{array}\right] \sim \begin{bmatrix} 1 & 0 & -2 & 1 & 0 & 0 \\ 0 & 1 & -2 & 3 & 1 & 0 \\ 0 & -3 & 8 & -2 & 0 & 1 \end{bmatrix} \sim \begin{bmatrix} 1 & 0 & -2 & 1 & 0 & 0 \\ 0 & 1 & -2 & 3 & 1 & 0 \\ 0 & 0 & 2 & 7 & 3 & 1 \end{bmatrix}$$

$$\sim \begin{bmatrix} 1 & 0 & -2 & 1 & 0 & 0 \\ 0 & 1 & 0 & 10 & 4 & 1 \\ 0 & 0 & 1 & 7/2 & 3/2 & 1/2 \end{bmatrix} \sim \left[\begin{array}{ccc|ccc} 1 & 0 & 0 & 8 & 3 & 1 \\ 0 & 1 & 0 & 10 & 4 & 1 \\ 0 & 0 & 1 & 7/2 & 3/2 & 1/2 \end{array}\right]$$

$\underbrace{\begin{bmatrix} 8 & 3 & 1 \\ 10 & 4 & 1 \\ 7/2 & 3/2 & 1/2 \end{bmatrix}}_{\text{is } A^{-1}}$ phew!

Can check $AA^{-1} = I_3$ & $A^{-1}A = I_3$.