MATH 22 WORKSHEET : Orthogonal sets & Projection

8/17/06
Barnett.

Let $\vec{u}_1 = \begin{bmatrix} 2 \\ 1 \\ -2 \end{bmatrix}$, $\vec{u}_2 = \begin{bmatrix} 1 \\ 0 \\ 1 \end{bmatrix}$, $\vec{u}_3 = \begin{bmatrix} -1 \\ 4 \\ 1 \end{bmatrix}$

A) Consider the subspace $W = \text{Span}\{\vec{u}_1, \vec{u}_2\}$. Is $\vec{u}_3$ in $W^\perp$?

B) Does $\{\vec{u}_1, \vec{u}_2, \vec{u}_3\}$ form an orthogonal set ? (how many tests did you do?)

C) Find, <u>without row reduction</u>, the coefficients of $\vec{y} = \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}$ in the basis $\{\vec{u}_1, \vec{u}_2, \vec{u}_3\}$:

$c_1 = \dots$ $\qquad$ $c_2 = \dots$ $\qquad$ $c_3 = \dots$

D) What is $\hat{y}$, the orthogonal projection of $\vec{y}$ onto $\vec{u}_2$?

$$\hat{y} = \begin{bmatrix} \cdot \\ \cdot \\ \cdot \end{bmatrix}$$

E) What is the (closest) distance from $\vec{y}$ to the line $\text{Span}\{\vec{u}_2\}$?

[Hint: draw a picture]

F) What is the orthogonal projection of $\vec{y}$ onto the subspace W ?
[Hint: is $\{\vec{u}_1, \vec{u}_2\}$ an orthog. basis for W?].

$$\hat{y} = \begin{bmatrix} \cdot \\ \cdot \\ \cdot \end{bmatrix}$$

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8/17/16

Let $\vec{u}_1 = \begin{bmatrix}2\\1\\-2\end{bmatrix}$, $\vec{u}_2 = \begin{bmatrix}1\\0\\1\end{bmatrix}$, $\vec{u}_3 = \begin{bmatrix}-1\\4\\1\end{bmatrix}$

A) Consider the subspace $W = \text{Span}\{\vec{u}_1, \vec{u}_2\}$. Is $\vec{u}_3$ in $W^\perp$?

$\vec{u}_1 \cdot \vec{u}_3 = -2+4-2 = 0$ $\quad \vec{u}_2 \cdot \vec{u}_3 = -1+0+1 = 0$ $\quad$ so $\vec{u}_3 \perp$ both vectors in Span of W.

$\Rightarrow$ yes.

B) Does $\{\vec{u}_1, \vec{u}_2, \vec{u}_3\}$ form an orthogonal set? (how many tests did you do?)

$\vec{u}_i \cdot \vec{u}_j = 0$ for all $i \neq j$. You've already tested $i=1, j=3$, $i=2, j=3$.

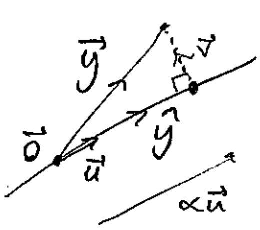
$\boxed{\text{3 tests, } \binom{n}{2} = 1+2+\dots+(n-1) \text{ in general.}}$

Only other combo is $\vec{u}_1 \cdot \vec{u}_2 = 2+0-2 = 0$ ✓ orthog. set.

C) Find, <u>without row reduction</u>, the coefficients of $\vec{y} = \begin{bmatrix}1\\2\\3\end{bmatrix}$ in the basis $\{\vec{u}_1, \vec{u}_2, \vec{u}_3\}$

$$c_1 = \frac{\overbrace{\vec{y}\cdot\vec{u}_1}^{2+2-6=-2}}{\underbrace{\vec{u}_1\cdot\vec{u}_1}_{2^2+1^2+2^2}} = -2/9 \qquad c_2 = \frac{\overbrace{\vec{y}\cdot\vec{u}_2}^{1+0+3=4}}{\underbrace{\vec{u}_2\cdot\vec{u}_1}_{1^2+0^2+1^2}} = 2 \qquad c_3 = \frac{\vec{y}\cdot\vec{u}_3}{\vec{u}_3\cdot\vec{u}_3} = 5/9$$

D) What is $\hat{y}$, the orthogonal projection of $\vec{y}$ onto $\vec{u}_2$?



$$\alpha = \frac{\vec{y}\cdot\vec{u}_2}{\vec{u}_2\cdot\vec{u}_2} = c_2 \text{ above} = 2$$

$$\hat{y} = \alpha\vec{u}_2 = 2\cdot\begin{bmatrix}1\\0\\1\end{bmatrix} \qquad \hat{y} = \begin{bmatrix}2\\0\\2\end{bmatrix}$$

E) What is the (closest) distance from $\vec{y}$ to the line $\text{Span}\{\vec{u}_2\}$?

here

[Hint: draw a picture]

$$\text{dist}(\vec{y}, \hat{y}) = \text{length of } \vec{v} = \|\vec{y} - \hat{y}\|$$

$$= \left\| \begin{bmatrix}1-2\\2-0\\3-2\end{bmatrix} \right\| = \sqrt{(-1)^2+2^2+1^2} = \sqrt{6}$$

F) What is the orthogonal projection of $\vec{y}$ onto the subspace W?

[Hint: is $\{\vec{u}_1, \vec{u}_2\}$ an orthog. basis for W?] (see Thm 8 in Sec. 6.3)

Can project separately onto each line $\vec{u}_1$, $\vec{u}_2$, then add them up.

$\alpha_1 = c_1$ above $= -2/9$ so $\hat{y}_1 = -2/9\begin{bmatrix}2\\1\\-2\end{bmatrix} = \begin{matrix}-4/9\\-2/9\\4/9\end{matrix}$ $\quad \hat{y} = \hat{y}_1 + \hat{y}_2 = \begin{bmatrix}-4/9+2\\-2/9+0\\4/9+2\end{bmatrix}$ $\quad \hat{y} = \begin{bmatrix}14/9\\-2/9\\22/9\end{bmatrix}$