MARKOV CHAIN PROBLEM

NOVEMBER 3, 2017

Problem 1. The weather in Washington is either good, indifferent, or bad on any given day. If the weather is good today, there is a 50% chance that the weather will be good tomorrow, 30% that it's indifferent, and 20% that it's bad. If it's indifferent today, it's good tomorrow with probability 20%, indifferent 80%. If bad today, it's good with probability 30% and indifferent 30%.

(a). What is the stochastic matrix, $\mathbf{P}$, for this situation?

(b). Find all the eigenvalues of $\mathbf{P}$.

(c). For each eigenvalue of $\mathbf{P}$, find the corresponding eigenvectors.

(d). What would be the long-run prediction for weather defined by

$$\mathbf{x}_{k+1} = \mathbf{P}\mathbf{x}_k, \qquad k = 0, 1, 2, \ldots$$

with $\mathbf{x}_0 = \begin{bmatrix} 0.45 \\ 0.45 \\ 0.1 \end{bmatrix}$?