BASIS WORKSHEET

OCTOBER 11, 2017

(1) Consider the following lemma.

Lemma. *Suppose A and B are $m \times n$ matrices such that $A = PB$ for some invertible matrix P. Then $A\mathbf{x} = \mathbf{0}$ iff $B\mathbf{x} = \mathbf{0}$ for any vector $\mathbf{x} \in \mathbb{R}^n$.*

Prove the lemma as follows.

(a) Suppose $\mathbf{x} \in \mathbb{R}^n$ with $B\mathbf{x} = \mathbf{0}$. Show that $A\mathbf{x} = \mathbf{0}$.

(b) Conversely, suppose $\mathbf{x} \in \mathbb{R}^n$ with $A\mathbf{x} = \mathbf{0}$. Show that $B\mathbf{x} = \mathbf{0}$. (*Hint*: Use P^{-1}.)

(2) Let

$$A = \begin{pmatrix} 1 & -2 & 2 & 6 \\ -2 & 4 & -3 & -9 \\ 3 & -6 & 3 & 9 \end{pmatrix}.$$

(a) Compute a basis for $\mathrm{Nul}(A)$.

(b) Compute a basis for $\mathrm{Col}(A)$.

(3) Show that the columns of the following matrix form a basis for $\mathbb{R}^3$. (*Hint*: Show that A is invertible.)

$$A = \begin{bmatrix} 1 & 2 & 1 \\ -2 & -4 & -6 \\ 1 & 1 & 4 \end{bmatrix}$$