

## SINGULAR VALUE DECOMPOSITION WORKSHEET

NOVEMBER 10, 2017

1. Compute an SVD of  $A = \begin{pmatrix} 3 & 2 \\ 2 & 3 \\ 2 & -2 \end{pmatrix}$ .

2. Use your answer to the previous question to find an SVD for  $A^T$  *without* having to recompute the decomposition from scratch.

3. (a) Show that if  $\mathbf{v}$  is an eigenvector of  $A^T A$  and  $A\mathbf{v} \neq \mathbf{0}$ , then  $A\mathbf{v}$  is an eigenvector of  $AA^T$  with the same eigenvalue.

(b) Show that if  $\mathbf{u}$  is an eigenvector of  $AA^T$  and  $A^T\mathbf{u} \neq \mathbf{0}$ , then  $A^T\mathbf{u}$  is an eigenvector of  $A^T A$  with the same eigenvalue.

(c) Conclude that  $A^T A$  and  $AA^T$  have the same nonzero eigenvalues.