

Math 22 Summer 2014 Midterm Exam I  
Friday July 11, 2014

PRINT NAME: \_\_\_\_\_

**INSTRUCTIONS: READ CAREFULLY!**

This is a closed book, closed notes exam. Use of calculators is not permitted. You have two hours, do all problems.

On all **free response** questions below you must show your step-by-step work and make sure it is clear *how* you arrived at your solution. Whenever you answer a question, don't just say "Yes" or "No", but always justify your answers. No credit is given for solutions without appropriate work or justification. You will receive partial credit for partially correct answers.

For multiple choice and True/False questions, no justification is necessary. Therefore, **leave no multiple choice question unanswered!** Guessing is allowed: A wrong guess does not cost you more points than leaving the question unanswered.

The Honor Principle requires that you neither give nor receive any aid on this exam.

GOOD LUCK!

FERPA waiver: By my signature I relinquish my FERPA rights in the following context: This exam paper may be returned en masse with others in the class and I acknowledge that I understand my score may be visible to others. If I choose not to relinquish my FERPA rights, I understand that I will have to present my student ID at my instructors office to retrieve my examination paper.

FERPA waiver signature:

Grader's use only:

1. \_\_\_\_\_ /13

2. \_\_\_\_\_ /10

3. \_\_\_\_\_ /15

4. \_\_\_\_\_ /10

5. \_\_\_\_\_ /15

6. \_\_\_\_\_ /7

7. \_\_\_\_\_ /20

8. \_\_\_\_\_ /10

**Total:** \_\_\_\_\_ /100

1. (a) [10 pt] Solve the following linear system:

$$\begin{array}{rcccccc} x_1 & & & - & 9x_3 & + & 2x_4 & = & -10 \\ -3x_1 & + & x_2 & + & 30x_3 & - & 7x_4 & = & 36 \\ 2x_1 & + & 3x_2 & - & 9x_3 & + & 2x_4 & = & -9 \end{array}$$

(b) [3 pt] Write the solution you found in parametric vector form, if possible.

2. Let  $T$  be the linear transformation given by

$$T(x_1, x_2, x_3) = (2x_2 + 10x_3, x_3, x_1 + 4x_2 + 5x_3, 2x_2 + 3x_3)$$

(a) [3 pt] What's the standard representing matrix for  $T$ ?

(b) [2 pt] What's the domain of  $T$ ? What's the codomain of  $T$ ?

(c) [5 pt] Is  $T$  onto? Is  $T$  one-to-one? Why or why not?

3. Consider the following matrix

$$A = \begin{pmatrix} 4 & -3 & -6 \\ -7 & 6 & 9 \\ -1 & 1 & 1 \end{pmatrix}$$

(a) [8 pt] Write the solution to the homogeneous system  $Ax = 0$  as the span of a set of vectors.

(b) [2 pt] Is the solution set of  $Ax = 0$  a point, line or plane? Explain your answer.

(c) [2 pt] Are the columns of  $A$  linearly independent? Why or why not?

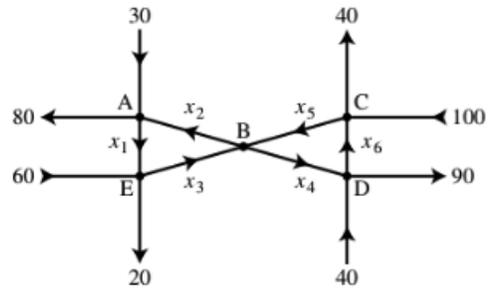
(d) [3 pt] Given that

$$\begin{pmatrix} 4 & -3 & -6 \\ -7 & 6 & 9 \\ -1 & 1 & 1 \end{pmatrix} \begin{pmatrix} 2 \\ 1 \\ -1 \end{pmatrix} = \begin{pmatrix} 11 \\ -17 \\ -2 \end{pmatrix}$$

find all the solutions to the equation  $Ax = b$  with  $b = \begin{pmatrix} 11 \\ -17 \\ -2 \end{pmatrix}$  **without solving the linear system**  $(A \ b)$ .

4. [10 pt]

Set up **and DO NOT SOLVE** the linear system associated to the following network flow.



5. [15 pt] Given the vectors  $v_1, v_2, v_3, b$  as follows

$$v_1 = \begin{pmatrix} 1 \\ -3 \\ 2 \\ 3 \end{pmatrix}, v_2 = \begin{pmatrix} 4 \\ -4 \\ 5 \\ 7 \end{pmatrix}, v_3 = \begin{pmatrix} 5 \\ -3 \\ 6 \\ 5 \end{pmatrix}, b = \begin{pmatrix} -1 \\ -7 \\ -1 \\ -2 \end{pmatrix}$$

is  $b$  in the span of the set  $\{v_1, v_2, v_3\}$ ?

6. [7 pt] Find all vectors  $b \in \mathbb{R}^2$  such that the system  $Ax = b$  has a solution, where  $A$  is the following matrix:

$$A = \begin{pmatrix} 3 & -1 \\ -9 & 3 \end{pmatrix}$$

7. **Multiple choice section:** each question is worth 5 points. No justification is needed, so don't leave any question blank.

(a) Consider the following transformation

$$T(x_1, x_2, x_3) = (x_3^2 - 5x_1, x_3 + 3x_1 - x_2, 0, 4x_3 - 2x_2).$$

Use the vectors in one the following sets (and ONLY those vectors) you can show that  $T$  is **NOT** a linear map. Which set is that?

$$(A) \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix} \quad (B) \begin{pmatrix} 1 \\ 3 \\ 0 \end{pmatrix}, \begin{pmatrix} 2 \\ 6 \\ 0 \end{pmatrix} \quad (C) \begin{pmatrix} 2 \\ 2 \\ 0 \end{pmatrix}, \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix}, \begin{pmatrix} 1 \\ 2 \\ 0 \end{pmatrix}$$
$$(D) \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix}, \begin{pmatrix} 0 \\ 0 \\ -1 \end{pmatrix} \quad (E) \begin{pmatrix} 1 \\ 0 \\ 2 \end{pmatrix}, \begin{pmatrix} 2 \\ 0 \\ 2 \end{pmatrix}$$

ANSWER

(b) Only one of the following sets of vectors is made of linearly independent vectors. Find that set.

$$(A) \begin{pmatrix} 1 \\ 5 \\ 7 \end{pmatrix}, \begin{pmatrix} 0 \\ 2 \\ 5 \end{pmatrix}, \begin{pmatrix} 7 \\ -3 \\ -2 \end{pmatrix}, \begin{pmatrix} -15 \\ 3 \\ 0 \end{pmatrix} \quad (B) \begin{pmatrix} -3 \\ 2 \\ 3 \end{pmatrix}, \begin{pmatrix} 2 \\ -\frac{4}{3} \\ -1 \end{pmatrix} \quad (C) \begin{pmatrix} 5 \\ 17 \\ -3 \end{pmatrix}, \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix}$$
$$(D) \begin{pmatrix} 2 \\ 9 \end{pmatrix}, \begin{pmatrix} -1 \\ -7 \end{pmatrix}, \begin{pmatrix} 5 \\ 9 \end{pmatrix}, \begin{pmatrix} -3 \\ -4 \end{pmatrix} \quad (E) \begin{pmatrix} -2 \\ 6 \\ -14 \end{pmatrix}, \begin{pmatrix} 1 \\ -3 \\ 7 \end{pmatrix}$$

ANSWER

(c) Suppose that

$$u = \begin{pmatrix} 7 \\ 2 \\ 5 \end{pmatrix}, v = \begin{pmatrix} 3 \\ 1 \\ 3 \end{pmatrix}, w = \begin{pmatrix} 5 \\ 1 \\ 1 \end{pmatrix}$$

and that you know that  $2u - w = 3v$ . Using this fact find  $x_1, x_2 \in \mathbb{R}$  that satisfy the equation

$$\begin{pmatrix} 7 & 3 \\ 2 & 1 \\ 5 & 3 \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \end{pmatrix} = \begin{pmatrix} 5 \\ 1 \\ 1 \end{pmatrix}$$

(A)  $\begin{pmatrix} \frac{2}{3} \\ \frac{1}{3} \end{pmatrix}$     (B)  $\begin{pmatrix} 2 \\ -1 \end{pmatrix}$     (C)  $\begin{pmatrix} -2 \\ 3 \end{pmatrix}$     (D)  $\begin{pmatrix} -2 \\ 1 \end{pmatrix}$     (E)  $\begin{pmatrix} 2 \\ -3 \end{pmatrix}$

ANSWER

(d) Suppose that you have a linear transformation  $T$  whose standard matrix has the following echelon form:

$$A = \begin{pmatrix} \bullet & * & * & * & * & * \\ 0 & 0 & \bullet & * & * & * \\ 0 & 0 & 0 & \bullet & * & * \\ 0 & 0 & 0 & 0 & 0 & \bullet \end{pmatrix}$$

where  $\bullet$  corresponds to a pivot position. Which of the following statements is true?

(A)  $T$  is one-to-one and onto;    (B)  $T$  is one-to-one but not onto;

(C)  $T$  is onto but not one-to-one;    (D)  $T$  is neither onto nor one-to-one;

ANSWER

### TRUE or FALSE?

For each of the statements below indicate whether it is true or false. You do not need to justify your answers. For each incorrect statement you'll lose 2 points. If five or more statements are incorrect you'll get 0 points out of the question.

8. [10 pt]

- (a) **True / False** A system of 3 equations in 4 unknowns always has a solution.
- (b) **True / False**  $Ax = b$  always has a solution if there is a pivot position in every row of  $A$ .
- (c) **True / False** The free variables in the solution of a linear system correspond to the pivot columns.
- (d) **True / False** If a system of linear equations has two solutions there must be infinitely many.
- (e) **True / False** A linear transformation  $T$  is onto if all rows of its standard matrix  $A$  have a pivot position.
- (f) **True / False** A set of one vector is always linearly independent.
- (g) **True / False** A system  $Ax = b$  is consistent if the last column of the augmented matrix  $(A \ b)$  is a pivot column.
- (h) **True / False** The solution set to  $Ax = b$  is  $p + v_h$ , where  $p$  is a solution and  $v_h$  is the general solution of  $Ax = 0$ .
- (i) **True / False** The matrix-vector product  $Ax$  is the linear combination of the columns of  $A$  with weights given by the entries of  $x$ .
- (j) **True / False** A linear transformation  $T$  is one-to-one if all rows of its standard matrix  $A$  have a pivot position.

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