Math 22, Exam I

April 22, 2010

NAME:

This is a closed book exam and you may not use a calculator. Use the space provided to answer the questions and if you need more space, please use the back of the exam making sure to write a note in the space provided that you have more work elsewhere that you would like me to grade. You must SHOW ALL WORK and be neat. If you have any questions, do not hesitate to ask.

Good luck!

Remember the honor code – do all of your own work.

1. Let

$$A = \begin{pmatrix} 5 & 8 & 7 \\ 0 & 1 & -1 \\ 1 & 3 & 0 \end{pmatrix} \qquad \text{and} \qquad \mathbf{b} = \begin{pmatrix} 2 \\ -3 \\ 2 \end{pmatrix}.$$

a. If consistent, solve the system $A\mathbf{x} = \mathbf{b}$ and write its solutions in parametric form. If it is not consistent, say so.

b. Solve the associated homogeneous system $A\mathbf{x} = \mathbf{0}$.

c. Is the system $A\mathbf{x} = \mathbf{c}$ consistent for all $\mathbf{c} \in \mathbb{R}^3$? Explain.

2. Consider the three vectors

$$\begin{pmatrix} 1 & -3 & -2 \\ 5 & h & -7 \end{pmatrix}$$

a. Find h such that the matrix is the augmented matrix of a consistent linear system.

b. Find h such that the three columns of the above matrix are linearly independent.

3. Compute the determinants of the following matrices:

a.

$$\begin{pmatrix} 5 & 4 \\ 3 & 2 \end{pmatrix}$$

b.

$$\begin{pmatrix} 0 & 1 & 1 \\ 1 & 3 & 4 \\ 2 & 2 & 3 \end{pmatrix}$$

c.

$$\begin{pmatrix} 4 & 0 & -7 & 3 & -5 \\ 0 & 0 & 2 & 0 & 0 \\ 7 & 3 & -6 & 4 & -8 \\ 5 & 0 & 5 & 2 & -3 \\ 0 & 0 & 9 & 0 & 2 \end{pmatrix}$$

4. Find the inverses of the following matrices.

a.

$$\begin{pmatrix} 5 & 4 \\ 3 & 2 \end{pmatrix}$$

b.

$$\begin{pmatrix} 0 & 1 & 1 \\ 1 & 3 & 4 \\ 2 & 2 & 3 \end{pmatrix}$$

c.

$$\begin{pmatrix} 1 & 2 & -1 \\ 5 & -2 & 4 \\ -1 & 2 & 3 \end{pmatrix}$$

5. Let

$$A = \begin{pmatrix} 3 & -7 & -2 \\ -3 & 5 & 1 \\ 6 & -4 & 0 \end{pmatrix}$$

a. Find the LU decomposition of A.

b. Let

$$B = \begin{pmatrix} 1 & 3 \\ 2 & 4 \end{pmatrix}.$$

Write B as a product of elementary matrices.

c Write B^{-1} as a product of elementary matrices.

6. Answer the following questions by true or false:

a. The inverse of an elementary matrix is an elementary matrix.

b. The following matrix is invertible

$$\begin{pmatrix} 0 & 1 & 1 & 1 & 0 \\ 1 & 0 & 0 & 0 & 1 \\ 1 & 1 & 1 & 1 & 1 \\ 0 & 2 & 0 & 3 & 0 \\ 4 & 5 & 6 & 7 & 8 \end{pmatrix}.$$

c. Any linear system of equations whose coefficient matrix is of type 3×4 has a free variable.

d. I like linear algebra.

7. Let $T : \mathbb{R}^3 \longmapsto \mathbb{R}^2$ be the linear map given by

$$T(x_1, x_2, x_3) = (3x_2 - x_3, 2x_1 + x_2 + 3x_3).$$

a. What is the domain of T?

b. What is the co-domain of T?

c. What is the standard matrix for T?

d. Is T onto? Why or why not?

e. Is T one-to-one? Why or why not?

8. Let $T : \mathbb{R}^2 \longmapsto \mathbb{R}^3$ be the linear transformation given by

$$T(\mathbf{e}_1) = \begin{pmatrix} 1 \\ 2 \\ 3 \end{pmatrix}, \qquad T(\mathbf{e}_2) = \begin{pmatrix} -1 \\ 0 \\ 4 \end{pmatrix}.$$

a. Compute

$$T \begin{pmatrix} 3 \\ 5 \end{pmatrix}.$$

b. Is $T : \mathbb{R}^3 \longmapsto \mathbb{R}^2$ given by $T(x_1, x_2, x_3) = (x_1 + 2x_3, x_1 + |x_2|)$ linear? Explain why or why not.

c. Suppose that A and B are $n \times n$ matrices such that both A and AB are invertible. Is B invertible?