Math 22, Summer 2013, Midterm II

Name (Print): ____

Last First

Instructions: You are not allowed to use calculators, books, or notes of any kind. You may not look at a classmate's exam for "inspiration." You must explain your reasoning behind each solution to receive full credit. Credit will not be awarded for correct answers with no explanation (with the exception of problem #1).

You may use pages 10-13 of the exam as scratch paper, but any work you intend to be graded should be on the exam itself in the space provided. If you run out of room, clearly indicate which page of scratch paper your solution is on and circle the solution that should be graded.

Before beginning the exam, skim through the problems to verify that you have one true/false question and five free-response questions.

Problem #	Points	Score
1	7	
2	7	
3	6	
4	9	
5	15	
6	6	
Total	50	

1. Determine whether each statement below is true or false and indicate your answer by circling the appropriate choice (1pt each):

(a) (True / False) Let A be an $m \times n$ matrix, and let B be an $n \times p$ matrix such that $AB = O$ (where O represents the $m \times p$ zero matrix). Then, the columns of B are in Nul$A$.

(b) (True / False) Let $\mathbb{P}_n$ denote the vector space of polynomials $p(x)$ of degree at most n. The set of all polynomials in $\mathbb{P}_n$ with $p(0) = 1$ is not a subspace of $\mathbb{P}_n$.

(c) (True / False) Suppose A is a 5×5 matrix with exactly 3 distinct eigenvalues. Suppose further that two eigenspaces of A are 2-dimensional. It is possible that A is not diagonalizable.

(d) (True / False) Suppose $B_3 = \begin{bmatrix} a & 0 & 0 \\ 0 & b & 1 \\ 0 & 0 & c \end{bmatrix}$ is an echelon form of A obtained through the following series of elementary row operations: B_1 is obtained by interchanging two rows of A; B_2 is obtained from B_1 by performing a row replacement; and lastly, a scaling of each row is performed so that $B_3 = \frac{1}{5}B_2$. Then $det(A) = -5^3abc$.

(e) (True / False) If $\mathbf{v}_1$ is an eigenvector of A corresponding to λ_1 and $\mathbf{v}_2$ is an eigenvector of A corresponding to λ_2, where λ_1 and λ_2 are distinct eigenvalues of A, then $\mathbf{v}_1$ and $\mathbf{v}_2$ are linearly independent.

(f) (True / False) Any linearly independent set in a subspace H is a basis for H.

(g) (True / False) If A is a 4×3 matrix whose null space has dimension 2, then A can have rank 2.

2. Let $A = \begin{bmatrix} 1 & 2 & 3 & 0 \\ 2 & 4 & 6 & 0 \\ 3 & 7 & 10 & 1 \end{bmatrix}$. Determine a basis for the following subspaces:

(a) Col$A$ (3pts)

(b) Row$A$ (2pts)

(c) Nul$A$ (2pts)

3. Suppose $H = Span\{\mathbf{e}_1\}$, $K = Span\{\mathbf{e}_2\}$, where $\{\mathbf{e}_1, \mathbf{e}_2\}$ is the standard basis for $\mathbb{R}^2$.

   (a) Explain why H and K are subspaces of $\mathbb{R}^2$. (2pts)

   (b) Is the intersection of H and K ($H \cap K$) a subspace of $\mathbb{R}^2$? Explain. (2pts)

   (c) Is the union of H and K ($H \cup K$) a subspace of $\mathbb{R}^2$? Explain. (2pts)

4. Determine whether the set $\mathrm{B} = \{p_1, p_2, p_3\}$ is a basis for $\mathbb{P}_2$ (the set of all polynomials of degree at most 2), where $p_1(x) = 3x^2 + x + 1$, $p_2(x) = 2x + 1$, and $p_3(x) = 2$. Fully justify your answer. (9pts)

5. (a) Suppose that an $n \times n$ matrix A has a zero eigenvalue. Explain why A must be a singular matrix. (1pt)

   (b) For the remaining parts of this problem, let $A = \begin{bmatrix} 1 & -1 \\ 2 & -2 \end{bmatrix}$. Determine the eigenvalue(s) of A. (2pts)

   (c) Determine a basis for each eigenspace of A. (4pts)

(d) Explain why A is diagonalizable. (1pt)

(e) Use the fact that A is diagonalizable to calculate A^{1000}. (7pts)

6. Find a basis for $H = \left\{ \begin{bmatrix} a+2b-4c \\ -5b+15c \\ a+b-c \\ a+b+3c \end{bmatrix} : a, b, c \in \mathbb{R} \right\}$. (6pts)