Math 22 –
Linear Algebra and its applications

- Lecture 1 -

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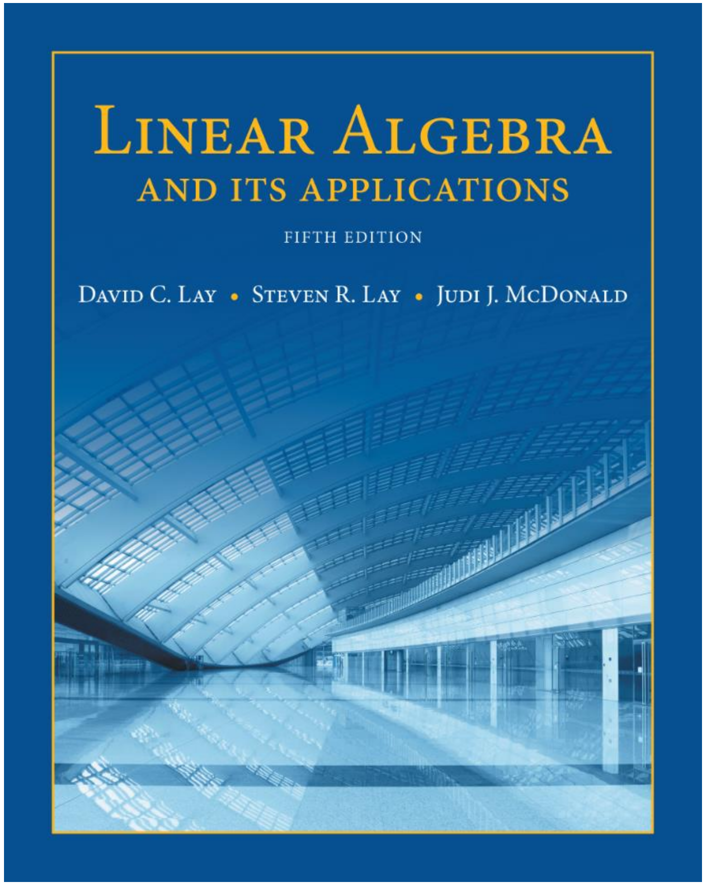
GENERAL INFORMATION

- **Office hours:** Tue 1-3 pm, Th, Sun 2-4 pm in KH 229
- **Tutorial:** Tue, Th, Sun 7-9 pm in KH 105
- **Workload:** Demanding course 10 -15 h / week is normal
- **Conclusion:** Engage yourself! Come to the class/tutorial, read the book, do homework and suggested practice problems until you feel confident with the topic.
- **No class** this Wednesday!

1 Linear Equations in Linear Algebra

1.1 SYSTEMS OF LINEAR EQUATIONS



GEOMETRIC INTERPRETATION

- **Example:**

GEOMETRIC INTERPRETATION

GEOMETRIC INTERPRETATION

LINEAR EQUATION

Aim: Learn an algorithm that can solve any system of linear equations or state that it has no solution.

LINEAR EQUATION

- A **linear equation** in the variables $x_1, \ldots, x_n$ is an equation that can be written in the form

$$a_1x_1 + a_2x_2 + \cdots + a_nx_n = b,$$

where b and the coefficients $a_1, \ldots, a_n$ are real or complex numbers that are usually known in advance.

- A **system of linear equations** (or a **linear system**) is a collection of one or more linear equations involving the same variables — say, $x_1, \ldots, x_n$.

LINEAR EQUATION

- A **solution** of the system is a list $(s_1, s_2, \ldots, s_n)$ of numbers that makes each equation a true statement when the values $s_1, \ldots, s_n$ are substituted for $x_1, \ldots, x_n$, respectively.
- The set of all possible solutions is called the **solution set** of the linear system.
- Two linear systems are called **equivalent** if they have the same solution set.

LINEAR EQUATION

- In the next lecture will see: A system of linear equations has

  1. no solution, or
  2. exactly one solution, or
  3. infinitely many solutions.

- A system of linear equation is said to be **inconsistent** if it has **no solution**.

- A system of linear equations is said to be **consistent** if it has either **one solution or infinitely many solutions**.

MATRIX NOTATION

- The essential information of a linear system can be recorded in a table or **matrix**.
- For the following system of equations,

$$\begin{aligned} x_1 - 2x_2 + x_3 &= 0 \\ 2x_2 - 8x_3 &= 8 \\ -4x_1 + 5x_2 + 9x_3 &= -9, \end{aligned}$$

the matrix
$$\begin{bmatrix} 1 & -2 & 1 \\ 0 & 2 & -8 \\ -4 & 5 & 9 \end{bmatrix}$$

is called the **coefficient matrix** of the system.

MATRIX NOTATION

- An **augmented matrix** of a system consists of the coefficient matrix with an added column containing the constants from the right sides of the equations.

- For the given system of equations,

$$\begin{bmatrix} 1 & -2 & 1 & 0 \\ 0 & 2 & -8 & 8 \\ -4 & 5 & 9 & -9 \end{bmatrix}$$

is called the **augmented matrix**.

MATRIX SIZE

- The **size of a matrix** tells how many rows and columns it has. If m and n are positive numbers, an $\boldsymbol{m \times n}$ **matrix** is a matrix or table with m rows and n columns. (The number of **rows** always comes **first**.)

- The **basic strategy** for solving a linear system is to replace one system with an equivalent system (*i.e.*, one with the same solution set) that is easier to solve.

SOLVING SYSTEM OF EQUATIONS

- **Example 1:** Solve the given system of equations.

$$x_1 - 2x_2 + x_3 = 0 \qquad \text{----(1)}$$
$$2x_2 - 8x_3 = 8 \qquad \text{----(2)}$$
$$-4x_1 + 5x_2 + 9x_3 = -9 \qquad \text{----(3)}$$

- **Solution:** Elimination algorithm using elementary row transformations.

SOLVING SYSTEM OF EQUATIONS

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- Thus, the only solution of the original system is (29,16,3). To verify that (29,16,3) is a solution, substitute these values into the left side of the original system, and compute.

$$(29)-2(16)+(3)=29-32+3=0$$
$$2(16)-8(3)=32-24=8$$
$$-4(29)+5(16)+9(3)=-116+80+27=-9$$

- The results agree with the right side of the original system, so (29,16,3) is a solution of the system.

ELEMENTARY ROW OPERATIONS

- Elementary row operations include the following:

  1. **(Replacement)** Replace one row by the sum of itself and a multiple of another row.
  2. **(Interchange)** Interchange two rows.
  3. **(Scaling)** Multiply all entries in a row by a nonzero constant.

- Two matrices are called **row equivalent** if there is a sequence of elementary row operations that transforms one matrix into the other.

ELEMENTARY ROW OPERATIONS

- It is important to note that **row operations** are **reversible**.

- If the **augmented matrices** of two linear systems are **row equivalent**, then the two systems have the **same solution set**.

- As already mentioned, a **central question** is whether the linear system has a **solution** and if the **solution** is **unique**.

EXISTENCE AND UNIQUENESS OF SYSTEM OF EQUATIONS

- **Example 2:** Determine if the following system is consistent.

$$\begin{aligned} x_2 - 4x_3 &= 8 \\ 2x_1 - 3x_2 + 2x_3 &= 1 \\ 5x_1 - 8x_2 + 7x_3 &= 1 \end{aligned} \qquad \text{----(4)}$$

- **Solution:** The augmented matrix is

$$\begin{bmatrix} 0 & 1 & -4 & 8 \\ 2 & -3 & 2 & 1 \\ 5 & -8 & 7 & 1 \end{bmatrix}$$

EXISTENCE AND UNIQUENESS OF SYSTEM OF EQUATIONS

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