MATH 22 HW 1
PLEASE SUBMIT ON GRADESCOPE AT ANY TIME BEFORE WEDNESDAY, SEPTEMBER 23 AT 5:59PM EDT

(1) (a) (5 points) Lay, Section 1.2, Problem 33. Note that there is only one solution in this particular case.

(b) (5 points) Can there be three points in $\mathbb{R}^2$ with distinct x coordinates which have more than one interpolating polynomial of degree 2? Either give an example or show this is impossible. (You might need to use facts outside linear algebra to do this! Hint: if two quadratics f, g pass through the same three points $(x_1, y_1), (x_2, y_2), (x_3, y_3)$, and h is the polynomial $f - g$, what is true about $h(x_i), i = 1, 2, 3$? Is this possible?).

(2) (10 points) Three vertices of a parallelogram are set at $(10, 22), (-3, 7), (2, -9)$. What are all of the possibilities for the fourth vertex? Explain how you came by your answers. (Hint: vector arithmetic.)

(3) (a) (5 points) Could a set of three vectors in $\mathbb{R}^4$ span all of $\mathbb{R}^4$? Explain. What about n vectors in $\mathbb{R}^m$ when n is less than m?

(b) (10 points) Without doing row reduction, determine which of the three sets of vectors below DO NOT span their respective $\mathbb{R}^n$ spaces. For the three which do not span, explain in a sentence or two how you arrived at your answer (you do not need to explain why the remaining sets do span).

(i) $\left\{ \begin{pmatrix} 1 \\ 0 \\ -1 \end{pmatrix}, \begin{pmatrix} 3 \\ 4 \\ 5 \end{pmatrix} \right\}$

(ii) $\left\{ \begin{pmatrix} 1 \\ 25 \\ 12 \end{pmatrix}, \begin{pmatrix} 1 \\ 25 \\ -12 \end{pmatrix}, \begin{pmatrix} 0 \\ 1 \\ 2 \end{pmatrix} \right\}$,

(iii) $\left\{ \begin{pmatrix} 1 \\ 2 \end{pmatrix}, \begin{pmatrix} -2 \\ -4 \end{pmatrix}, \begin{pmatrix} -4 \\ -8 \end{pmatrix} \right\}$,

(iv) $\left\{ \begin{pmatrix} 0 \\ 1 \\ 1 \\ 0 \end{pmatrix}, \begin{pmatrix} 1 \\ 1 \\ 0 \\ 0 \end{pmatrix}, \begin{pmatrix} 0 \\ 0 \\ 1 \\ 1 \end{pmatrix}, \begin{pmatrix} 0 \\ 0 \\ 0 \\ -5 \end{pmatrix} \right\}$,

(v) $\left\{ \begin{pmatrix} 1 \\ 0 \\ 2 \\ 2 \\ 1 \end{pmatrix}, \begin{pmatrix} 2 \\ 0 \\ -3 \\ -3 \\ 8 \end{pmatrix}, \begin{pmatrix} 98 \\ 0 \\ 1 \\ 0 \\ -6 \end{pmatrix}, \begin{pmatrix} 7 \\ 0 \\ 9 \\ 1 \\ 3 \end{pmatrix}, \begin{pmatrix} 9 \\ 0 \\ 4 \\ -10 \\ 6 \end{pmatrix} \right\}$,

(vi) $\left\{ \begin{pmatrix} 0 \\ 2 \end{pmatrix}, \begin{pmatrix} 2 \\ 0 \end{pmatrix} \right\}$.