MATH 22 HW 4
PLEASE SUBMIT ON GRADESCOPE AT ANY TIME BEFORE WEDNESDAY, OCTOBER 21 AT 5:59PM EDT

To earn full credit, show all work and explain your answers carefully. Make your arguments using complete sentences. Technology should not be used for linear algebra computations. Exceptions: you may use technology to invert, multiply, and row-reduce matrices.The graders will take away 1 point for every question submission on Gradescope that is not properly tagged.

(1) (a) (5 points) Lay, Section 4.5, 6

(b) (5 points) Lay, Section 4.5, 22 and 44.

(c) (5 points) Give an explicit example of a 3-dimensional subspace $W \subset \mathbb{R}^4$ such that e_2 and e_3 are elements of W but e_1 and e_4 are not, or prove that such a subspace does not exist. If you give an example, make sure to explain how you know that your subspace has all of the desired properties (i.e., that it is a subspace, that it has dimension 3, and that it contains and does not contain the appropriate vectors).
(Hint: Is there a 2-dimensional subspace of $\mathbb{R}^3$ that contains e_1 but not e_2 or e_3?)

(2) (a) (5 points) Suppose that U and W are two subspaces of a vector space V, and that $U \cap W = \{\vec{0}\}$. Let $\{u_1; \ldots u_k\}$ be a list of linearly independent vectors in U and $\{w_1; \ldots w_p\}$ be a list of linearly independent vectors in W. Show that $\{u_1; \ldots u_k; w_1; \ldots w_p\}$ is a linearly independent list.

(b) (5 points) Show that if $\dim U + \dim W > \dim V$, then $U \cap W \neq \{\vec{0}\}$.

(3) (5 points) Suppose V is a finite dimensional vector space, and let $T\colon V \to V$ be a linear transformation. Show that T is into if and only if T is onto. (Note: V is not necessarily $\mathbb{R}^n$ so you should NOT cite the Invertible Matrix Theorem! There is another theorem which might be more helpful, though.)

(4) (5 points) Let $V = \{f \in \mathbb{P}_7 \mid f(2) = f(3) = 0\}$ (i.e., the set of polynomials of degree at most 7 which have roots at 2 and 3). Construct a linear transformation with domain $\mathbb{P}_7$ for which V is the kernel. What is the dimension of V? Do not use row reduction.