

**Math 22**  
Homework # 4

Write careful solutions for the homework that demonstrates a command of what you have learned on week #4. Do not carry out computations without telling the reader why you are doing the computation. If you say something is true provide a short explanation using definitions or Theorems. Hand-in something that you can feel proud of.

1. Provide careful explanations to the following questions.

- (a) Is it possible for a  $5 \times 5$  matrix to be invertible when its columns do not span  $\mathbb{R}^5$ ?
- (b) If  $C$  is a  $6 \times 6$  matrix and the equation  $C\mathbf{x} = \mathbf{v}$  is consistent for all  $\mathbf{v} \in \mathbb{R}^6$ , is it possible for  $C\mathbf{x} = \mathbf{v}$  to have more than one solution for some  $\mathbf{v}$ .
- (c) If  $H$  is a square matrix and the equation  $H\mathbf{x} = \mathbf{c}$  is inconsistent for some  $\mathbf{v}$ , what can you say about the equation  $H\mathbf{x} = \mathbf{0}$ ?

2. Let  $A = \begin{bmatrix} 1 & 2 & -5 & 11 & -1 \\ 2 & 4 & -5 & 15 & 2 \\ 1 & 2 & 0 & 4 & 5 \\ 3 & 6 & -5 & 19 & -2 \end{bmatrix}$  be row equivalent to  $B = \begin{bmatrix} 1 & 2 & 0 & 4 & 5 \\ 0 & 0 & 5 & -7 & 8 \\ 0 & 0 & 0 & 0 & -9 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}$

- (a) Find a basis for Nul  $A$  and find its dimension.
  - (b) Find a basis for Col  $A$  and find the rank of  $A$ .
3. Let  $T : V \rightarrow W$  be a linear transformation. Show that if  $\{\mathbf{v}_1, \dots, \mathbf{v}_p\}$  is a linearly dependent subset of  $V$ , then  $\{T(\mathbf{v}_1), \dots, T(\mathbf{v}_p)\}$  is a linearly dependent subset of  $W$ .
4. Determine if the following sets are linearly independent. Explain why or why not. If not find a basis for the space spanned by the set.
- (a)  $\{1 + t, 1 - t, 2\}$ , so the set contains three polynomials  $\mathbf{p}_1(t) = 1 + t$ ,  $\mathbf{p}_2(t) = 1 - t$  and  $\mathbf{p}_3(t) = 2$ .
  - (b)  $\{1 + t^2, 1 - t^2\}$ , this set contains two polynomials  $\mathbf{p}_1(t) = 1 + t^2$  and  $\mathbf{p}_2(t) = 1 - t^2$ .