

**Math 22**  
Homework 6

Write careful solutions for the homework that demonstrates a command of what you have learned on week #6. Do not carry out computations without telling the reader why you are doing the computation. If you say something is true provide a short explanation using definitions or Theorems. Hand-in something that you can feel proud of.

1. Let  $\lambda$  be an eigenvalue of an invertible matrix  $A$ . Show that  $\lambda^{-1}$  is an eigenvalue of  $A^{-1}$ .
2. Show that If  $\lambda$  is an eigenvalue of  $A$ , then it is also an eigenvalue of  $A^T$ .
3. It can be shown that the algebraic multiplicity of an eigenvalue is always greater than or equal to the dimension of the corresponding eigenspace. Find  $h$  in the matrix  $A$  below so that the eigenspace for  $\lambda = 5$  is two dimensional.

$$A = \begin{bmatrix} 5 & -2 & 6 & -1 \\ 0 & 3 & h & 0 \\ 0 & 0 & 5 & 4 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

4. Diagonalize  $A = \begin{bmatrix} -7 & -16 & 4 \\ 6 & 13 & -2 \\ 12 & 16 & 1 \end{bmatrix}$  given that one eigenvalue is  $\lambda = 5$  and one eigenvector is  $\mathbf{b}_1 = \begin{bmatrix} -2 \\ 1 \\ 2 \end{bmatrix}$ .