Math 22

Homework 8

Write careful solutions for the homework that demonstrates a command of what you have learned on week #8. Do not carry out computations without telling the reader why you are doing the computation. If you say something is true provide a short explanation using definitions or Theorems. Hand-in something that you can feel proud of.

1. Use Gram-Schmidt to produce an orthogonal basis for the span of $\left\{ \begin{bmatrix} 3 \\ -1 \\ 2 \\ -1 \end{bmatrix}, \begin{bmatrix} -5 \\ 9 \\ -9 \\ 3 \end{bmatrix} \right\}$.

2. Find an orthogonal basis for the column space of the matrix $\begin{bmatrix} -1 & 6 & 6 \\ 3 & -8 & 3 \\ 1 & -2 & 6 \\ 1 & -4 & -3 \end{bmatrix}$.

3. Orthogonally diagonlize the matrix $\begin{bmatrix} 4 & 0 & 1 & 0 \\ 0 & 4 & 0 & 1 \\ 1 & 0 & 4 & 0 \\ 0 & 1 & 0 & 4 \end{bmatrix}$ giving an orthogonal matrix P and a diagonal matrix D. The eigenvalues are given: they are 3, 5.

4. A laboratory animal may eat any one of three foods each day. Laboratory records show that if the animal chooses one food on one trial, it will choose the same food on the next trial with a probability of 50%, and it will choose the other foods on the next trial with equal probabilities of 25%.

   (a) What is the stochastic matrix for this situation?

   (b) If the animal chooses food #1 on the initial trial, what is the probability that it will choose food # 2 on the second trial after the initial trial? (Note there are two trials after the initial trial).

   (c) Will this matrix have a unique steady-state vector? Explain why or why not?

   (d) Find the steady-state vector. Which food will the animal prefer after many trials?