Math 22
Homework 1

Instructions: In this homework you should make sure that you write careful solutions with details explaining what you are doing. This is your opportunity to demonstrate what you have learned in the first week of classes. Although there are other methods for solving some of the problems (for example problems # 1), to receive credit use the theorems and algorithms that we learned in class. Upload only pdf files, here is a link to a tutorial on how to upload homework https://www.youtube.com/watch?v=u-pK4GzpId0.

1. Solve the system:
$$
\begin{array}{rcrcrcr}
x_1 & - & 3x_2 & + & 4x_3 & = & -4 \\
3x_1 & - & 7x_2 & + & 7x_3 & = & -8 \\
-4x_1 & + & 6x_2 & - & x_3 & = & 7
\end{array}
$$

2. Suppose each matrix represents the augmented matrix of a linear system (boxes are pivot positions and the stars any number). Determine if the systems are consistent or not and if consistent, whether the solution is unique or not.

   (a) $\begin{bmatrix} \blacksquare & * & * \\ 0 & \blacksquare & * \\ 0 & 0 & 0 \end{bmatrix}$

   (b) $\begin{bmatrix} \blacksquare & * & * & * & * \\ 0 & 0 & \blacksquare & * & * \\ 0 & 0 & 0 & \blacksquare & * \end{bmatrix}$.

3. Give the parametric form of the solution of the linear system that has augmented matrix:
$$
\begin{bmatrix}
1 & 2 & -5 & -6 & 0 & -5 \\
0 & 1 & -6 & -3 & 0 & 2 \\
0 & 0 & 0 & 0 & 1 & 0 \\
0 & 0 & 0 & 0 & 0 & 0
\end{bmatrix}
$$

4. Determine if $\mathbf{b} = \begin{bmatrix} -5 \\ 11 \\ -7 \end{bmatrix}$ is a linear combination of the vectors
$$
\mathbf{a}_1 = \begin{bmatrix} 1 \\ -2 \\ 2 \end{bmatrix}, \quad \mathbf{a}_2 = \begin{bmatrix} 0 \\ 5 \\ 5 \end{bmatrix}, \quad \mathbf{a}_3 = \begin{bmatrix} 2 \\ 0 \\ 8 \end{bmatrix}.
$$