

Math 22
Homework 2

Write careful solutions for the homework that demonstrates a command of what you have learned on week #2. Do not carry out computations without telling the reader why you are doing the computation. If you say something is true provide a short explanation using definitions or Theorems. Hand-in something that you can feel proud of.

1. Answer the following questions and provide an explanation. The explanation is more important than the answer.
 - (a) Suppose A is a 3×3 matrix with two pivot positions. Does the equation $A\mathbf{x} = \mathbf{0}$ have a non-trivial solution? Does the equation $A\mathbf{x} = \mathbf{b}$ have at least one solution for every possible \mathbf{b} ?
 - (b) Suppose A is a 2×4 matrix with two pivot positions. Does the equation $A\mathbf{x} = \mathbf{0}$ have a non-trivial solution? Does the equation $A\mathbf{x} = \mathbf{b}$ have at least one solution for every possible \mathbf{b} ?
2. Solve the problem making sure to tell me why you are doing the computations and why you are making your conclusions. Suppose you have the following vectors:

$$\mathbf{v}_1 = \begin{bmatrix} 1 \\ -5 \\ -3 \end{bmatrix}, \quad \mathbf{v}_2 = \begin{bmatrix} -2 \\ 10 \\ 6 \end{bmatrix}, \quad \mathbf{v}_3 = \begin{bmatrix} 2 \\ -9 \\ h \end{bmatrix}.$$

- (a) For what values of h is \mathbf{v}_3 in $\text{Span}\{\mathbf{v}_1, \mathbf{v}_2\}$?
 - (b) For what values of h is $\{\mathbf{v}_1, \mathbf{v}_2, \mathbf{v}_3\}$ linearly dependent?
3. Let W be the following set:

$$W = \left\{ \begin{bmatrix} s + 3t \\ s - t \\ 2s - t \\ 4t \end{bmatrix} \text{ such that } s, t \in \mathbb{R} \right\}$$

- (a) Use the definition of subspace to show that W is a subspace of \mathbb{R}^4 .
 - (b) Use the theorem about the span of any set of vectors being a subspace to show that W is a subspace.
4. Let

$$A = \begin{bmatrix} -8 & -2 & -9 \\ 6 & 4 & 8 \\ 4 & 0 & 4 \end{bmatrix}, \quad \mathbf{w} = \begin{bmatrix} 2 \\ 1 \\ -2 \end{bmatrix}$$

- (a) Is \mathbf{w} in $\text{Col } A$? Explain.
- (b) Is \mathbf{w} in $\text{Nul } A$? Explain.