Math 22

Homework # 5

Write careful solutions for the homework that demonstrates a command of what you have learned on week #5. Do not carry out computations without telling the reader why you are doing the computation. If you say something is true provide a short explanation using definitions or Theorems. Hand-in something that you can feel proud of.

1. Define $T : \mathbb{P}_3 \to \mathbb{R}^4$ by $T(\mathbf{p}) = \begin{bmatrix} \mathbf{p}(-3) \\ \mathbf{p}(-1) \\ \mathbf{p}(1) \\ \mathbf{p}(3) \end{bmatrix}$.

   (a) Show that T is a linear transformation.

   (b) Find the matrix of T relative to $\mathcal{B} = \{1, t, t^2, t^3\}$ and $\mathcal{E}$ the standard basis for $\mathbb{R}^4$

2. In $\mathbb{P}_2$ find the change of coordinates matrix from the basis $\mathcal{B} = \{1-3t^2, 2+t-5t^2, 1+2t\}$ to the standard basis $\mathcal{C} = \{1, t, t^2\}$. Then write t^2 as a linear combination of the polynomials in $\mathcal{B}$.

3. Let $\mathcal{D} = \{\mathbf{d}_1, \mathbf{d}_2, \mathbf{d}_3\}$ and $\mathcal{F} = \{\mathbf{f}_1, \mathbf{f}_2, \mathbf{f}_3\}$ be bases for a vector space V, and suppose that $\mathbf{f}_1 = 2\mathbf{d}_1 - \mathbf{d}_2 + \mathbf{d}_3$, $\mathbf{f}_2 = 3\mathbf{d}_2 + \mathbf{d}_3$, and $\mathbf{f}_3 = -3\mathbf{d}_1 + 2\mathbf{d}_3$.

   (a) Find the change of coordinates matrix from $\mathcal{F}$ to $\mathcal{D}$, that is ${}_{\mathcal{D}}[Id]_{\mathcal{F}}$.

   (b) Find the matrix ${}_{\mathcal{F}}[Id]_{\mathcal{D}}$.

   (c) Find $[\mathbf{x}]_{\mathcal{D}}$ for $\mathbf{x} = \mathbf{f}_1 - 2\mathbf{f}_2 + 2\mathbf{f}_3$.

4. Compute the determinant using the method asked.

   (a) Use cofactor expansions making sure to choose at each step the row or column that involves the least amount of computation.

$$\det(A) = \begin{vmatrix} 6 & 3 & 2 & 4 & 0 \\ 9 & 0 & -4 & 1 & 0 \\ 8 & -5 & 6 & 7 & 1 \\ 2 & 0 & 0 & 0 & 0 \\ 4 & 2 & 3 & 2 & 0 \end{vmatrix}.$$

   (b) Use row reduction to echelon form.

$$\det(A) = \begin{vmatrix} 1 & 3 & 2 & -4 \\ 0 & 1 & 2 & -5 \\ 2 & 7 & 6 & -3 \\ -3 & -10 & -7 & -2 \end{vmatrix}$$