

Math 22
Homework 1

Instructions: In this homework you should make sure that you write careful solutions with details explaining what you are doing. This is your opportunity to demonstrate what you have learned in the first week of classes. Although there are other methods for solving some of the problems (for example problems # 1), to receive credit use the theorems and algorithms that we learned in class. Upload only pdf files, here is a link to a tutorial on how to upload homework <https://www.youtube.com/watch?v=u-pK4GzpId0>.

1. Solve the following system using the row reduction algorithm.

$$\begin{aligned}x_1 - 3x_2 + 4x_3 &= -4 \\3x_1 - 7x_2 + 7x_3 &= -8 \\-4x_1 + 6x_2 - x_3 &= 7\end{aligned}$$

2. Suppose each matrix represents the augmented matrix of a linear system (boxes are pivot positions and the stars any number). Determine if the systems are consistent or not and if consistent, whether the solution is unique or not.

(a) $\left[\begin{array}{ccc|c} \blacksquare & * & * & * \\ 0 & \blacksquare & * & * \\ 0 & 0 & 0 & * \end{array} \right]$

(b) $\left[\begin{array}{ccccc|c} \blacksquare & * & * & * & * & * \\ 0 & 0 & \blacksquare & * & * & * \\ 0 & 0 & 0 & \blacksquare & * & * \end{array} \right]$.

3. Give the parametric form of the solution of the linear system that has augmented matrix:

$$\left[\begin{array}{ccccc|c} 1 & 2 & -5 & -6 & 0 & -5 \\ 0 & 1 & -6 & -3 & 0 & 2 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{array} \right]$$

4. Let $\mathbf{a}_1 = \begin{bmatrix} 1 \\ -2 \\ 2 \end{bmatrix}$, $\mathbf{a}_2 = \begin{bmatrix} 0 \\ 5 \\ 5 \end{bmatrix}$, $\mathbf{a}_3 = \begin{bmatrix} 2 \\ 0 \\ 8 \end{bmatrix}$ and $\mathbf{b} = \begin{bmatrix} -5 \\ 11 \\ -7 \end{bmatrix}$.

- (a) Write a matrix-vector equation that is equivalent to the problem of determining if \mathbf{b} is a linear combination of \mathbf{a}_1 , \mathbf{a}_2 , and \mathbf{a}_3 .
- (b) Either show that \mathbf{b} cannot be expressed as a linear combination, or find weights that express it as such.
5. A steam plant burns two types of coal: anthracite (A) and bituminous (B). For each ton of A burned, the plant produces 27.6 million Btu of heat, 3100 grams (g) of sulfur dioxide, and 250 g of particulate matter (solid particle pollutants). For each ton of B burned, the plant produces 30.2 million Btu, 6400 g of sulfur dioxide, and 360 g of particulate matter.

- (a) How much heat does the steam plant produce when it burns x_1 tons of A and x_2 tons of B ?
- (b) Suppose the output of the steam plant is described by a vector that lists the amount of heat, sulfur dioxide, and particulate matter. Express this output as a linear combination of two vectors, assuming that the plant burns x_1 tons of A and x_2 tons of B .