Math 22

Homework 2

Write careful solutions for the homework that demonstrates a command of what you have learned on week #2. Do not carry out computations without telling the reader why you are doing the computation. If you say something is true provide a short explanation using definitions or Theorems. Hand-in something that you can feel proud of.

1. Answer the following questions and provide an explanation. The explanation is more important than the answer.

   (a) Suppose A is a 3×3 matrix with two pivot positions. Does the equation $A\mathbf{x} = \mathbf{0}$ have a non-trivial solution? Does the equation $A\mathbf{x} = \mathbf{b}$ have at least one solution for every possible $\mathbf{b}$?

   (b) Suppose A is a 2×4 matrix with two pivot positions. Does the equation $A\mathbf{x} = \mathbf{0}$ have a non-trivial solution? Does the equation $A\mathbf{x} = \mathbf{b}$ have at least one solution for every possible $\mathbf{b}$?

2. Solve the problem making sure to tell me why you are doing the computations and why you are making your conclusions. Suppose you have the following vectors:

$$\mathbf{v}_1 = \begin{bmatrix} 1 \\ -5 \\ -3 \end{bmatrix}, \qquad \mathbf{v}_2 = \begin{bmatrix} -2 \\ 10 \\ 6 \end{bmatrix}, \qquad \mathbf{v}_3 = \begin{bmatrix} 2 \\ -9 \\ h \end{bmatrix}.$$

   (a) For what values of h is $\mathbf{v}_3$ in $\text{Span}\{\mathbf{v}_1, \mathbf{v}_2\}$?

   (b) For what values of h is $\{\mathbf{v}_1, \mathbf{v}_2, \mathbf{v}_3\}$ linearly dependent?

3. Let W be the following set:

$$W = \left\{ \begin{bmatrix} s + 3t \\ s - t \\ 2s - t \\ 4t \end{bmatrix} : s, t \in \mathbb{R} \right\}$$

   (a) Use the definition of subspace to show that W is a subspace of $\mathbb{R}^4$.

   (b) Use the theorem about the span of any set of vectors being a subspace to show that W is a subspace.

4. Let

$$A = \begin{bmatrix} -8 & -2 & -9 \\ 6 & 4 & 8 \\ 4 & 0 & 4 \end{bmatrix}, \qquad \mathbf{w} = \begin{bmatrix} 2 \\ 1 \\ -2 \end{bmatrix}$$

   (a) Is $\mathbf{w}$ is in $\text{Col}\, A$? Explain.

(b) Is $\mathbf{w}$ in Nul A ? Explain.

5. If a mass m is placed at the end of a spring, and if the mass is pulled downward and released, the mass-spring system will begin to oscillate. The displacement $y(t)$ of the mass from its resting position is given by a function of the form

$$y(t) = a \cos \omega t + b \sin \omega t \tag{1}$$

where ω is a constant that depends on the spring and the mass (recall that $\sin \omega t$ and $\cos \omega t$ are periodic functions – they repeat as a sinusoidal curve with periodicity $\frac{2\pi}{\omega}$ – this repetition is mirrored in the back and forth movement of a perfect spring that never loses energy.)

Show that the set of all functions described in Equation (1) (with ω fixed and a, b arbitrary numbers) is a vector space. These vector spaces are important for communications systems where sound and data travel on sinusoidal waves.