Math 22

Homework # 4

Write careful solutions for the homework that demonstrates a command of what you have learned on week #4. Do not carry out computations without telling the reader why you are doing the computation. If you say something is true provide a short explanation using definitions or Theorems. Hand-in something that you can feel proud of.

1. Provide careful explanations to the following questions.

   (a) Is it possible for a 5×5 matrix to be invertible when its columns do not span $\mathbb{R}^5$?

   (b) If C is a 6×6 matrix and the equation $C\mathbf{x} = \mathbf{v}$ is consistent for all $\mathbf{v} \in \mathbb{R}^6$, is it possible for $C\mathbf{x} = \mathbf{v}$ to have more than one solution for some $\mathbf{v}$.

   (c) If H is a square matrix and the equation $H\mathbf{x} = \mathbf{v}$ is inconsistent for some $\mathbf{v}$, what can you say about the equation $H\mathbf{x} = \mathbf{0}$?

2. Let $A = \begin{bmatrix} 1 & 2 & -5 & 11 & -1 \\ 2 & 4 & -5 & 15 & 2 \\ 1 & 2 & 0 & 4 & 5 \\ 3 & 6 & -5 & 19 & -2 \end{bmatrix}$ be row equivalent to $B = \begin{bmatrix} 1 & 2 & 0 & 4 & 5 \\ 0 & 0 & 5 & -7 & 8 \\ 0 & 0 & 0 & 0 & -9 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}$

   (a) Find a basis for Nul A and find its dimension.

   (b) Find a basis for Col A and find the rank of A.

3. Let $T : V \to W$ be a linear transformation. Show that if $\{\mathbf{v}_1, \ldots, \mathbf{v}_p\}$ is a linearly dependent subset of V, then $\{T(\mathbf{v}_1), \ldots, T(\mathbf{v}_p)\}$ is a linearly dependent subset of W.

4. Determine if the following sets are linearly independent. Explain why or why not. If not find a basis for the space spanned by the set.

   (a) $\{1 + t, 1 - t, 2\}$, so the set contains three polynomials $\mathbf{p}_1(t) = 1 + t$, $\mathbf{p}_2(t) = 1 - t$ and $\mathbf{p}_3(t) = 2$.

   (b) $\{1 + t^2, 1 - t^2\}$, this set contains two polynomials $\mathbf{p}_1(t) = 1 + t^2$ and $\mathbf{p}_2(t) = 1 - t^2$.