Math 22

Homework 6

Write careful solutions for the homework that demonstrates a command of what you have learned on week #6. Do not carry out computations without telling the reader why you are doing the computation. If you say something is true provide a short explanation using definitions or Theorems. Hand-in something that you can feel proud of.

1. Let λ be an eigenvalue of an invertible matrix A. Show that λ^{-1} is an eigenvalue of A^{-1}.

2. Show that If λ is an eigenvalue of A, then it is also an eigenvalue of A^T.

3. It can be shown that the algebraic multiplicity of an eigenvalue is always greater than or equal to the dimension of the corresponding eigenspace. Find h in the matrix A below so that the eigenspace for $\lambda = 5$ is two dimensional.

$$A = \begin{bmatrix} 5 & -2 & 6 & -1 \\ 0 & 3 & h & 0 \\ 0 & 0 & 5 & 4 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

4. Diagonalize $A = \begin{bmatrix} -7 & -16 & 4 \\ 6 & 13 & -2 \\ 12 & 16 & 1 \end{bmatrix}$ given that one eigenvalue is $\lambda = 5$ and one eigenvector is $\mathbf{b}_1 = \begin{bmatrix} -2 \\ 1 \\ 2 \end{bmatrix}$.

5. For the matrices below, diagonalize if possible. If not possible, explain why.

   (a) $\begin{bmatrix} 1 & 0 \\ 6 & -1 \end{bmatrix}$

   (b) $\begin{bmatrix} 4 & 2 & 2 \\ 2 & 4 & 2 \\ 2 & 2 & 4 \end{bmatrix}$ (For this matrix the eigenvalues are $\lambda = 2, 8$)