Math 22

Homework 7

Write careful solutions for the homework that demonstrates a command of what you have learned on week #7. Do not carry out computations without telling the reader why you are doing the computation. If you say something is true provide a short explanation using definitions or Theorems. Hand-in something that you can feel proud of.

1. Show that $\{\mathbf{u}_1, \mathbf{u}_2, \mathbf{u}_3\} = \left\{ \begin{bmatrix} 3 \\ -3 \\ 0 \end{bmatrix}, \begin{bmatrix} 2 \\ 2 \\ -1 \end{bmatrix}, \begin{bmatrix} 1 \\ 1 \\ 4 \end{bmatrix} \right\}$ is an orthogonal basis for $\mathbb{R}^3$ and write $\mathbf{x} = \begin{bmatrix} 5 \\ -3 \\ 1 \end{bmatrix}$ as a linear combination of the $\mathbf{u}_i$.

2. Let $W = \text{Span}\{\mathbf{u}_1, \mathbf{u}_2, \mathbf{u}_3\}$ where $\mathbf{u}_1 = \begin{bmatrix} 1 \\ 1 \\ 0 \\ -1 \end{bmatrix}$, $\mathbf{u}_2 = \begin{bmatrix} 1 \\ 0 \\ 1 \\ 1 \end{bmatrix}$, and $\mathbf{u}_3 = \begin{bmatrix} 0 \\ -1 \\ 1 \\ -1 \end{bmatrix}$. Write $\mathbf{y} = \begin{bmatrix} 3 \\ 4 \\ 5 \\ 6 \end{bmatrix}$ as the sum of a vector in W and a vector in $W^{\perp}$.

3. Let $\mathbf{y} = \begin{bmatrix} 7 \\ 9 \end{bmatrix}$, $\mathbf{u}_1 = \begin{bmatrix} 1/\sqrt{10} \\ -3/\sqrt{10} \end{bmatrix}$, and $W = \text{Span}\{\mathbf{u}_1\}$.
   Let $U = \begin{bmatrix} 1/\sqrt{10} \\ -3/\sqrt{10} \end{bmatrix} = [\mathbf{u}_1]$ viewed as a 2×1 matrix.
   (a) Compute $U^T U$ and UU^T.
   (b) Compute $\text{proj}_W \mathbf{y}$ using the definition and $(UU^T)\mathbf{y}$

4. Find the best approximation to $\mathbf{z} = \begin{bmatrix} 2 \\ 4 \\ 0 \\ -1 \end{bmatrix}$ by vectors of the form $c_1\mathbf{v}_1 + c_2\mathbf{v}_2$ where $\mathbf{v}_1 = \begin{bmatrix} 2 \\ 0 \\ -1 \\ -3 \end{bmatrix}$ and $\mathbf{v}_2 = \begin{bmatrix} 5 \\ -2 \\ 4 \\ 2 \end{bmatrix}$.

5. Find the equation $y = ct + d$ of the least squares line that best fits the following data points: $(1, 0), (2, 1), (4, 2), (5, 3)$