Your name:

Instructor (please circle): Alex Barnett Michael Musty

Math 22 Summer 2017, Homework 2, due Fri July 7 *Please show your work, and check your answers. No credit is given for solutions without work or justification.*

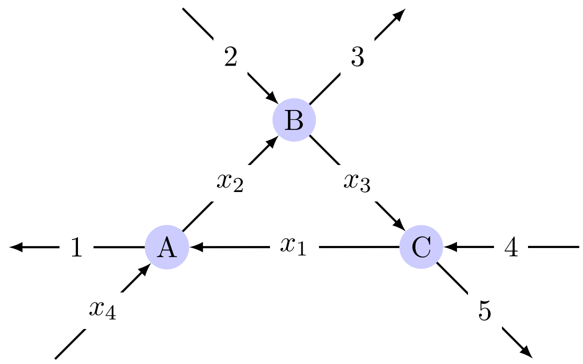
(1) Let h be a scalar, and consider the set of three vectors:

$$\mathbf{v}_1 = \begin{bmatrix} 3 \\ -2 \\ -4 \end{bmatrix}, \quad \mathbf{v}_2 = \begin{bmatrix} -6 \\ 3 \\ 9 \end{bmatrix}, \quad \mathbf{v}_3 = \begin{bmatrix} -3 \\ 2(h-1) \\ 7 \end{bmatrix}.$$

(a) With $h = 1/2$, is the set linearly independent, and why? If not, give a dependence relation between $\mathbf{v}_1$, $\mathbf{v}_2$, and $\mathbf{v}_3$.

(b) With $h = 0$, is the set linearly independent, and why? If not, give a dependence relation between $\mathbf{v}_1$, $\mathbf{v}_2$, and $\mathbf{v}_3$.

(2) Consider the following network flow diagram where the numbers indicate known flows, and x_1, x_2, x_3, x_4 indicate unknown flows on their respective edges.



(a) Find the *reduced* echelon form for the corresponding linear system.

(b) If a solution exists, write the *parametric vector form* of the solution set.

(c) Assume all flows are nonnegative. What constraint does this impose on x_3?

(3) (a) Let $T : \mathbb{R}^3 \to \mathbb{R}^3$ be a linear transformation defined by

$$T(\mathbf{x}) = x_1\mathbf{v}_1 + x_2\mathbf{v}_2 + x_3\mathbf{v}_3, \quad \text{with}$$

$$\mathbf{x} = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}, \quad \mathbf{v}_1 = \begin{bmatrix} 3 \\ -2 \\ 5 \end{bmatrix}, \quad \mathbf{v}_2 = \begin{bmatrix} 4 \\ -3 \\ 1 \end{bmatrix}, \text{ and } \quad \mathbf{v}_3 = \begin{bmatrix} 7 \\ 1 \\ 2 \end{bmatrix}.$$

Find a matrix A such that $T(\mathbf{x}) = A\mathbf{x}$ for every $\mathbf{x}$ in $\mathbb{R}^3$. Explain why your answer is correct.

(b) Let $\mathbf{p}$ be any vector in $\mathbb{R}^n$ and let $\mathbf{v}$ be any nonzero vector in $\mathbb{R}^n$. Consider the line ℓ defined by the parametric equation

$$\mathbf{x} = \mathbf{p} + t\mathbf{v}, \quad t \text{ in } \mathbb{R}.$$

Let $T : \mathbb{R}^n \to \mathbb{R}^n$ be a linear transformation. Show that the image of ℓ under the map T is either a line in $\mathbb{R}^n$ or a single point.