Your name:

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Math 22 Summer 2017, Homework 8, due Fri August 18 *Please show your work, and check your answers. No credit is given for solutions without work or justification.*

(1) Let $A = \begin{bmatrix} 5 & 1 & -1 & 5 \\ 1 & 5 & 5 & -1 \\ -5 & 1 & 1 & 5 \end{bmatrix}$ and note that the rows of A are orthogonal.

(a) Without using row reduction, write $\mathbf{y} = \begin{bmatrix} 11 \\ 5 \\ 3 \\ -1 \end{bmatrix}$ as a linear combination of the rows of A.

(b) Let $W = \mathrm{Col}\,(A^T)$. To which fundamental subspace of the matrix A is $W^{\perp}$ equal?

(c) What is $\dim W^{\perp}$? Prove your answer.

(2) Let $\mathbf{v}_1 = \begin{bmatrix} 1 \\ 1 \\ -2 \end{bmatrix}$, $\mathbf{v}_2 = \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}$, $\mathbf{y} = \begin{bmatrix} -1 \\ 1 \\ 3 \end{bmatrix}$. Let $W = \text{Span}\{\mathbf{v}_1, \mathbf{v}_2\}$.

(a) Find the element of W whose distance to $\mathbf{y}$ is as small as possible.

(b) Compute the distance from the previous part.

(c) Let U be the 3×2 matrix whose columns are $\mathbf{v}_1/\|\mathbf{v}_1\|$ and $\mathbf{v}_2/\|\mathbf{v}_2\|$. *Without* computing any matrix-vector multiplication, find $UU^T\mathbf{y}$ and explain why.

(d) Without computing a matrix-vector multiplication, compute the 2-norm of the vector $U \begin{bmatrix} 4 \\ 3 \end{bmatrix}$, with U as in part (c). Explain.

(3) Let $A = \begin{bmatrix} -1 & 0 & 0 \\ 1 & 1 & 0 \\ 0 & 1 & -2 \\ 0 & 0 & 1 \end{bmatrix}$.

(a) Find an orthogonal basis for Col A using the Gram-Schmidt algorithm.

(b) Compute the QR factorization of A. (i.e. Find matrices Q and R so that $A = QR$, the columns of Q form an orthonormal basis for Col A, and R is an upper triangular invertible matrix with positive entries along its diagonal.)

BONUS what happens during Gram-Schmidt if the columns of A were linearly dependent?