Your name:

Instructor (please circle): Alex Barnett Michael Musty

Math 22 Summer 2017, Homework 9, due start of Wed Aug 23 class

This one is shorter since you have 2 days less.

(1) Linear regression! Let x_1 be the intercept and x_2 be the slope for a general linear function $y(t) = x_1 + x_2 t$. Find its *least squares fit* to the data $(0, 0)$, $(2, -2)$, and $(3, 4)$, which are three points (t, y) in the plane. Here's how to set up the linear system (you don't need to read Sec. 6.6 unless interested): The first point says $x_1 + x_2.0 = 0$, the next says $x_1 + x_2.2 = -2$, and the last says $x_1 + x_2.3 = 4$.

(a) The system is inconsistent. Find the least squares solution vector(s) $\hat{\mathbf{x}} = \begin{bmatrix} \hat{x}_1 \\ \hat{x}_2 \end{bmatrix}$.

You've now learned the best-fit intercept $\hat{x}_1$ and slope $\hat{x}_2$! Is this solution unique?

(b) Let A be *any* matrix, possibly rectangular. Prove that if $A^T A$ is invertible, then the columns of A are linearly independent.

(2) (a) Consider the matrix $A = \begin{bmatrix} 2 & 1 & 0 \\ 1 & 0 & 1 \end{bmatrix}$. Over all vectors $\mathbf{x}$ in $\mathbb{R}^3$ with $\|\mathbf{x}\| = 1$, what is the largest $\|A\mathbf{x}\|$ can be? [Hint: if it helps, exploit that that AA^T and $A^T A$ have identical *nonzero* eigenvalues.]

(b) Compute by hand the full SVD of the previous A, ie give U, Σ, and V. [Hints: find the third column of V however you like, and make sure that your $\mathbf{u}_j$ vectors match your $\mathbf{v}_j$ vectors in ordering and sign]