Barnett
7/31/17

Your name: ~ SOLUTIONS ☺ ~

10 pts total.

Math 22 Summer 2017, mini-quiz 2, Mon July 31

Please show your work. No credit is given for solutions without work or justification.

[3pts] (1) Compute the determinant of $A = \begin{bmatrix} 1 & 0 & 1 & 0 \\ 2 & 0 & 5 & 6 \\ 3 & 0 & 1 & 1 \\ 4 & 2 & 1 & 1 \end{bmatrix}$.

cofactor expansion about col 2.
signs $-, +, -, +$

$$\det A = +2 \begin{vmatrix} 1 & 1 & 0 \\ 2 & 5 & 6 \\ 3 & 1 & 1 \end{vmatrix}$$

expand 1st row (since has a zero).

$$= 2\left(1 \begin{vmatrix} 5 & 6 \\ 1 & 1 \end{vmatrix} - 1 \begin{vmatrix} 2 & 6 \\ 3 & 1 \end{vmatrix}\right)$$

$$= 2\left(5(1) - 6(1) - \left(2(1) - 6(3)\right)\right)$$

$$= 2(15) = 30.$$

Note merely stating rows of A (or cols) are a basis, without proof, is inadequate.

[another version: $\det A \neq 0$ so by IMT, rows of A are L.I. since cols of A^T are, since A^T invertible. Since rows of A span Row A, they are a basis.

[2pts] (2) Use the result from question 1 to find a basis for Row A, where A is the matrix from question 1, *without* row reducing:

$\det A \neq 0$, so A is invertible, so A has full rank (all pivots, ie 4 of them), by IMT.

Thus Row A has dimension 4, so is equal to $\mathbb{R}^4$.

A basis is thus $\{\vec{e}_1, \vec{e}_2, \vec{e}_3, \vec{e}_4\}$, the standard basis for $\mathbb{R}^4$.

Alternatively, A row equivalent to I_4, so rows of I_4 (or columns) are a basis for Row A.

It's also true that the rows (or cols) of A are a basis for Row A

(3) Let $H = \left\{ \begin{bmatrix} a \\ b \\ c \end{bmatrix} : 2a + b - c = 0 \right\}$. ← Here a, b, c are real numbers.

[2pts] (a) Prove H is a vector space. [Hint: there is no time nor space to test the basic axioms] ↑all.

$H = \text{Nul } A$, for $A = [2 \ \ 1 \ \ -1]$, a 1×3 matrix.

From class, any null space is a subspace, here of $\mathbb{R}^3$, hence is also a vector space.

[2pts] (b) Find a basis for H and state dim H.

$A \sim [1 \ \ 1/2 \ \ -1/2]$, x_2 free, x_3 free.

so $\vec{x} = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} -\frac{1}{2}x_2 + \frac{1}{2}x_3 \\ x_2 \\ x_3 \end{bmatrix}$

basis is $\left\{ \begin{bmatrix} -1/2 \\ 1 \\ 0 \end{bmatrix}, \begin{bmatrix} 1/2 \\ 0 \\ 1 \end{bmatrix} \right\}$, $\dim H = 2$.

[1pt] ↑ since already recently tested

(c) Every point in H is in the span of the set $\mathbf{e}_1$, $\mathbf{e}_2$, $\mathbf{e}_3$ (the unit vectors in $\mathbb{R}^3$). Is this set a basis for H, and why?

No: i) the set is linearly independent, but

ii) $\text{Span}\{\vec{e}_1, \vec{e}_2, \vec{e}_3\} = \mathbb{R}^3 \neq H$.

the span is larger than H, not equal to H as a set.

A good way to prove the latter is by example:

$\begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}$ is in $\text{Span}\{\vec{e}_1, \vec{e}_2, \vec{e}_3\}$ but not in H, since $2(1)+0-0 \neq 0$.