
MATH 22 LECTURE 07 CLASSWORK: ANSWERS

JULY 05, 2017

For each described operation, find the standard matrix A and determine if T is onto and/or one-to-one.

(1) Let $T : \mathbb{R}^n \rightarrow \mathbb{R}^m$ be defined by $T(x_1, x_2) = (3x_1, -2x_1 + x_2, -x_2)$.

(a) What is n ? $n = 2$

(b) What is m ? $m = 3$

(c) What is A ? $A = \begin{bmatrix} 3 & 0 \\ -2 & 1 \\ 0 & -1 \end{bmatrix}$ whose echelon form has 2 pivots.

(d) Is T onto? Is there a pivot in every row? No, so the map is not onto.

(e) Is T one-to-one? Is there a pivot in every column? Yes, so the map is one-to-one.

(2) Let $T : \mathbb{R}^2 \rightarrow \mathbb{R}^2$ be reflection about the line $x_2 = x_1$.

(a) What is A ? $A = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$

(b) Is T onto? Is there a pivot in every row? Yes, so the map is onto.

(c) Is T one-to-one? Is there a pivot in every column? Yes, so the map is one-to-one.

(3) Let $T : \mathbb{R}^3 \rightarrow \mathbb{R}^2$ be defined by $(x_1, x_2, x_3) \mapsto (x_1, x_2)$.

(a) What is A ? $A = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \end{bmatrix}$

(b) Is T onto? Is there a pivot in every row? Yes, so the map is onto.

(c) Is T one-to-one? Is there a pivot in every column? No, so the map is not one-to-one.