Lecture 14

Math 22 Summer 2017
July 19, 2017

Just for today

- §4.3 Bases of a vector space
- Midterm1 tonight 6pm - 8pm in Kemeny 008

§4.3 Definition of basis

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A concise way to think about a basis is as a *minimal spanning set*.

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- Let $\mathcal{B} = \{\mathbf{v}_1, \mathbf{v}_2, \mathbf{v}_3\}$ be a set of 3 vectors in $\mathbb{R}^3$.

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- Let $\mathcal{B} = \{\mathbf{v}_1, \mathbf{v}_2, \mathbf{v}_3\}$ be a set of 3 vectors in $\mathbb{R}^3$.
  How can we check if $\mathcal{B}$ is a basis?

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What's the proof?

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$$A = \begin{bmatrix} 1 & 0 & 3 & 0 & 11 & 0 & 0 \\ 0 & 1 & 2 & 0 & 7 & 0 & 0 \\ 0 & 0 & 0 & 1 & 5 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 13 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}.$$

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What changes if A is not in RREF?

§4.3 Example continued

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Now consider the matrix B whose RREF is A.

$$B = \begin{bmatrix} 1 & 0 & 3 & 0 & 11 & 0 & 0 \\ 0 & 1 & 2 & 0 & 7 & -3 & -39 \\ -1 & 0 & -3 & 0 & -11 & 1 & 13 \\ 0 & -2 & -4 & 0 & -14 & 6 & 78 \\ 0 & 0 & 0 & 1 & 5 & 0 & 0 \end{bmatrix}, \quad A = \begin{bmatrix} 1 & 0 & 3 & 0 & 11 & 0 & 0 \\ 0 & 1 & 2 & 0 & 7 & 0 & 0 \\ 0 & 0 & 0 & 1 & 5 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 13 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}.$$

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Let's organize these observations in a theorem...

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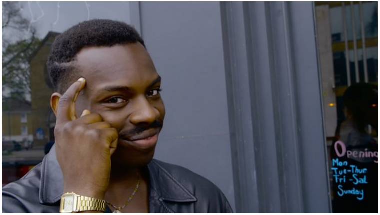

§4.3 Classwork

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- If $\{\mathbf{v}_1, \mathbf{v}_2, \mathbf{v}_3, \mathbf{v}_4\}$ is a linearly dependent set, then $\mathbf{v}_4 \in \text{Span}\{\mathbf{v}_1, \mathbf{v}_2, \mathbf{v}_3\}$.

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- A linear transformation $T : \mathbb{R}^n \to \mathbb{R}^m$ is onto if for each $\mathbf{x} \in \mathbb{R}^n$, there is a $\mathbf{b} \in \mathbb{R}^m$ such that $T(\mathbf{x}) = \mathbf{b}$. **False**
- A, B invertible $\implies ((AB)^{-1})^T = (A^T)^{-1}(B^T)^{-1}$. **True**
- The solutions of a linear system are changed by row operations. **False**
- T is onto if every column of $[T]$ has a pivot. **False**
- Any set containing the zero vector is linearly dependent. **True**
- For $A\mathbf{x} = \mathbf{0}$ to have a solution, A must have a pivot in every row. **False**
- If $\{\mathbf{v}_1, \mathbf{v}_2, \mathbf{v}_3, \mathbf{v}_4\}$ is a linearly dependent set, then $\mathbf{v}_4 \in \text{Span}\{\mathbf{v}_1, \mathbf{v}_2, \mathbf{v}_3\}$. **False**

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