Lecture 19

Math 22 Summer 2017
July 31, 2017

Just for today

- §5.1 Finish up
- §5.2 Characteristic polynomials

§5.1 Theorem 1

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Proof.

Let $A = (a_{ij})$. λ is an eigenvalue of A if and only if the null space of $A - \lambda I_n$ contains a nonzero vector. Write out $A - \lambda I_n$ under the assumption that A is triangular to show that $(A - \lambda I_n)\mathbf{x} = \mathbf{0}$ has a free variable precisely when $\lambda = a_{kk}$ for some $k \leq n$. □

§5.1 Theorem 2

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We will prove this by contradiction.

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$$\boxed{\mathbf{v}_{p+1} = c_1\mathbf{v}_1 + \cdots + c_p\mathbf{v}_p}, \quad \text{with } \{\mathbf{v}_1, \ldots, \mathbf{v}_p\} \text{ independent.}$$

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Multiply by A on both sides to get

$$A\mathbf{v}_{p+1} = Ac_1\mathbf{v}_1 + \cdots + Ac_p\mathbf{v}_p = c_1A\mathbf{v}_1 + \cdots + c_pA\mathbf{v}_p.$$

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How can we use the boxed equations to get a contradiction? □

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λ is an eigenvalue of A precisely when $(A - \lambda I_n)\mathbf{x} = \mathbf{0}$ has a nontrivial solution. This is equivalent to $A - \lambda I_n$ *not* being invertible. But $A - \lambda I_n$ is invertible precisely when $\det(A - \lambda I_n) \neq 0$. □

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Let $P^{-1}AP = B$. Then

$$\det(B - \lambda I_n) = \det\left(P^{-1}(A - \lambda I_n)P\right) = \det(P^{-1})\det(A - \lambda I_n)\det(P).$$

□

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- $\mathbf{x}_k = A^k\Big(\underbrace{(3/8)\mathbf{v}_1 + (9/40)\mathbf{v}_2}_{\mathbf{x}_0}\Big) = (3/8)\lambda_1^k\mathbf{v}_1 + (9/40)\lambda_2^k\mathbf{v}_2$

§5.2 Classwork

Find the eigenvalues of the following matrices:

1. $A = \begin{bmatrix} 1 & 6 \\ 5 & 2 \end{bmatrix}$
2. $A = \begin{bmatrix} 1 & -1 \\ 1 & 1 \end{bmatrix}$
3. $A = \begin{bmatrix} -5 & 0 \\ 0 & -5 \end{bmatrix}$
4. $A = \begin{bmatrix} 1 & 3 & 3 \\ -3 & -5 & -3 \\ 3 & 3 & 1 \end{bmatrix}$
5. $A = \begin{bmatrix} -7 & 9 & 0 \\ 0 & 1 & 3 \\ 0 & 0 & 1 \end{bmatrix}$

§5.2 Classwork Solutions

Find the eigenvalues of the following matrices:

1. $A = \begin{bmatrix} 1 & 6 \\ 5 & 2 \end{bmatrix}$, $\text{charpoly}(A) = (\lambda + 4)(\lambda - 7)$
2. $A = \begin{bmatrix} 1 & -1 \\ 1 & 1 \end{bmatrix}$, $\text{charpoly}(A) = \lambda^2 - 2\lambda + 2$
3. $A = \begin{bmatrix} -5 & 0 \\ 0 & -5 \end{bmatrix}$, $\text{charpoly}(A) = (\lambda + 5)^2$
4. $A = \begin{bmatrix} 1 & 3 & 3 \\ -3 & -5 & -3 \\ 3 & 3 & 1 \end{bmatrix}$, $\text{charpoly}(A) = (\lambda - 1)(\lambda + 2)^2$
5. $A = \begin{bmatrix} -7 & 9 & 0 \\ 0 & 1 & 3 \\ 0 & 0 & 1 \end{bmatrix}$, $\text{charpoly}(A) = (\lambda + 7)(\lambda - 1)^2$

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What is the dimension of the $\lambda = -2$ eigenspace?

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What is the dimension of the $\lambda = -2$ eigenspace? 2.

Both eigenvalues have **algebraic multiplicity** 2 but their **geometric multiplicities** are not equal.