Math 23 Diff Eq: Homework 2

due Wed Oct 10

Problems from Boyce & DiPrima, given in order in which we covered material. (Remember to show your working/reasoning—answers without explanation will not receive a high score!)

2.4: 13 (hint: lecture Mon Sept 26), 20 (be precise with verbal description, and/or use a sketch), 22 ab (I really care about part b), 32 [1].

2.6: 2, 4, 13 (hint try to get an explicit form for $y(x)$)

2.7: Numerical solution of $y' = 1 + t - y$

Use the following code `euler.m` (get from website) or something similar, to get an approximate solution, given $y(0) = 2$:

```
% The Euler method, (c) L. Euler, 1768.
f = @(t,y) 1+t-y;                                % set up function f(t,y)
t0 = 0; y0 = 2;                                  % IC
h = 0.1;                                         % time step
T = 4;                                           % final (stopping) time

N = (T-t0)/h;                                    % number of steps
clear ys ts                                      % empty the vectors
ys(1) = y0;                                      % first y,t given by IC
ts(1) = t0;                                      % (NB indexing starts at 1)
for n=1:N
  ys(n+1) = ys(n) + h*f(ts(n),ys(n));            % Euler update for y
  ts(n+1) = ts(n) + h;                           % fill the time values too
end
```

Now you may want to study and adapt commands from the end of `intro.m` [Hint: keep a text file of your commands and paste into the Matlab window as needed].

1. Plot a graph of this numerical solution using `+` signs. Add to this plot, using lines, the exact solution (which you'll need to find algebraically, then add a line of code to compute!) Label your axes.
2. Plot the *difference* between the numerical and exact solutions. What magnitude is the worst error you see?
3. Repeat with `h` ten times smaller. Roughly by what factor do errors shrink? Using this, *estimate* how big `N` would need to be to get errors of less than 10^{-6}

2.3: 3 (connects to 2.4.32), 14 (this could represent a seasonal variation in birth rate, as happens with many animals. Use `ode45` in Matlab, as in `intro.m`)

2.5: 3, 7 (introduces a new concept, note k in a is not same as k above. Hint for c: change variable), 22 (Hint: partial fractions)

[1] For electrical engineers and physicists, this is an RC low-pass filter driven by a single square voltage pulse! Why? Can you see what the value of $\tau = RC$ is?