

Math 23 Diff Eq: Homework 7

due Wed Nov 14 ... but best if do relevant questions after each lecture

You will do a little Matlab again this time—I have tried to maximize the intuition it gives you but minimize pain (little new coding, but try it early and ask if stuck). In problem “A” you solve the familiar damped oscillator but from a *ODE system* viewpoint. Why bother doing this numerically when you already did it analytically? Because most real-world problems are *nonlinear* and *not analytically solvable*: such numerical methods are then among your only friends.

7.4: 2abc,

4 (remember $x_2^{(1)}$, or x_{21} , is second element of first solution vector. This question shows you 2nd-order and 1st-order-system Wronskians are just facets of the same thing!),

6 (b means to say ‘in what time intervals’).

7.5: 2,

13 (Hint: you could check your eigen-calculation by entering the matrix into Matlab with $\mathbf{A} = \begin{bmatrix} \mathbf{a} & \mathbf{b} & \mathbf{c}; \\ \mathbf{d} & \mathbf{e} & \mathbf{f}; \\ \mathbf{g} & \mathbf{h} & \mathbf{k} \end{bmatrix}$ then $[\mathbf{V}, \mathbf{D}] = \text{eig}(\mathbf{A})$, giving (normalized) eigenvectors in columns of \mathbf{V} and eigenvalues on diagonal of \mathbf{D}),

16,

25.

7.6: 1 (important to be able to do this),

17.

7.8: 1 (use `pplane7`),

2.

9.1: 4 (sketch $x_1(t)$ by hand by looking at `pplane7` output),

17 (reviews mass-spring using your new geometric language. The next but one problem will help you to check your answers. By the way, nothing electric is required so don’t do that bit),

19.

A. Matlab’s Runge-Kutta solver `ode45` can also handle systems of ODEs, by feeding it a *column vector* initial condition, here use $\mathbf{x}_0 = [1; 0]$, and a column vector function, *e.g.* to solve the equation $\mathbf{x}' = \mathbf{A}\mathbf{x}$ with general 2-by-2 matrix¹ use the (*t*-independent) function $\mathbf{f} = \text{@(t,x)} \mathbf{A}*\mathbf{x}$. Use this (cannibalizing `intro.m` or your HW2) to numerically solve the above mass-spring system for $m = 1$, $\gamma = 0.1$, $k = 10$, then plot x vs t in $0 < t < 100$. Then do a 3D plot of $\mathbf{x}(t)$, that is (x, y, t) . If you got your output vectors \mathbf{xs} via `[ts, xs] = ode45(...)`, then you’ll want

```
plot3(xs(:,1), xs(:,2), ts); axis vis3d; xlabel('x'); ylabel('y'); zlabel('t');
```

Rotate (click box symbol then drag plot) from all angles until you grasp its shape—cool, eh?

¹What’s A in your case? Remember it will need to be defined *before* \mathbf{f} is.