Math 23 Diff Eq: Homework 8

due Wed Nov 28 . . . therefor a bit longer—I strongly suggest you do half before you leave for break

Note on integrals: these days it's professional to check your integrals symbolically, especially since you have a bunch to do to get Fourier coefficients. You could use Matlab's Symbolic Toolbox (separate licence but Dartmouth may have). Or here's example commands in (free) Maple to compute $\int_{-L}^{L} x \sin(n\pi x/L)\, dx$.

```
assume(n,integer);
f := x*sin(n*x*Pi/L);
A := int(f,x=-L..L);
```

Gives answer $2(-1)^{n+1}L^2/n\pi$. How great is that? Not required for our course–this is purely to help you out!

9.2: 3 (follow Example 1; this is another way of getting whether CW or CCW),

6 (use `pplane7` or applet),

19 a (you may do b for fun but don't need to hand in).

10.1: 1,

5,

8 (the last two also review your 2nd-order linear ODE methods),

14,

16.

10.2: 8,

10 (both easy),

13 (sawtooth wave),

14 (welcome to your first Fourier series!),

19 (see Example in 10.3. For the plot you can use the Fourier applet on square-wave setting; you don't need to write Matlab code).

A. Let's derive the orthogonality properties of sin and cos on p. 578. First evaluate $\int_{-1/2}^{1/2} e^{2\pi iky}dy$ for k integer (consider $k = 0$ too). Now write sin and cos using Euler's formula. Then expand $\int_{-1/2}^{1/2} \cos(2\pi ny)\cos(2\pi my)dy$ using Euler's formula, and treat the 4 terms using your first result. Finally, change variable $x = 2Ly$. Repeat for the other two orthogonality integrals. This will be relatively painless.

10.3: 2 (Consider the Theorem when drawing the sketch. Watch out for the way series is written in back; you will find expressions such as $\cos n\pi = (-1)^n$ for integer n useful),

17.

10.4: 1,

6,

7,

27 (for c & d, you don't need to plot. Instead just answer d by comparing triangle vs sawtooth waves on the applet, or comparing Fig. 10.2.4 and Fig. 10.4.3).