Math 23 Diff Eq: Homework 9

due Wed Dec 5
Shorter to leave time to review for the final, which is 2 days later

There are lots of very cool matlab PDE simulations you could do. Instead for simplicity's sake I have opted for two tasks which you can perform with applets alone.

10.5: 9.

13 (You don't need to plot anything here: instead bound the ratio of the n^{th} term in the sum to the first term, and solve for the n value that gives you a ratio of about 0.001. Note, you might get a transcendental equation to solve: you can avoid this by ignoring the $1/n$ dependence of the b_n and focusing only on the exponential decay in t, which is a good approximation)

10.6: 1 (easy)

9 abd (Part a is crucial. For the rest, don't use the computer, just sketch the plots using

`http://www.math.cornell.edu/ bterrell/h1/heat1.html`

where you can enter ICs and BCs and watch it animate. For d give an approximation by answering: how much time must elapse before the value of the $n = 1$ sine series term at $x = 5$ drops to 1% of the steady-state value at $x = 5$? This will give a simple algebraic equation to solve. α^2 for aluminum is given on p. 604).

12 ab (Ask yourself: is the $\sin(\pi x/L)$ initial condition an eigenfunction of the $X(x)$ ODE? If not, what are the eigenfunctions? Hint: orthogonality between sin and cos does *not* apply here since not a full 2π cycle is being integrated over)

10.7: 1 ab (Use the `falstad.com/loadedstring` applet with maximum number of 'loads', on 'pluck string' mode. Adjust animation speed until you can see it slow enough to sketch at a few times. State your t values in terms of fractions of a period of the motion, eg $t = 1/8$ period etc).

10.8: 1 ab.

8 a (this requires a slight variant on the rectangular separation of variables you just did. Just write the general solution and choose the part which has the desired decay as $y \to \infty$).

Congratulations! You've done a lot of new and valuable solution techniques this semester!