Existence & Uniqueness for ODEs

What do you need to test?

Nonlinear

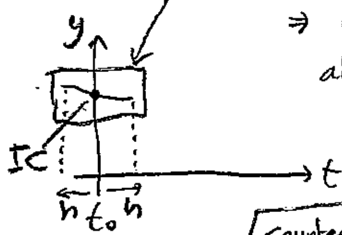
1st-order

put in standard form $y' = f(t,y)$, $y(t_0) = y_0$

Thm 2.4.2:

f and $\frac{\partial f}{\partial y}$ cont. in neighborhood of IC

$\Rightarrow$ unique soln exist at least for some time $h \geq 0$ about t_0



$\boxed{\text{counterexample: } y' = y^{1/3}}$

2nd-order

N/A

1st-order systems

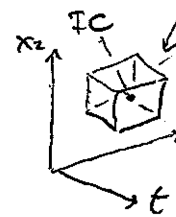
standard form $\begin{cases} x_1' = f_1(t, x_1, \dots x_n) \\ \vdots \\ x_n' = f_n(t, x_1, \dots x_n) \end{cases}$

& ICs.

Thm 7.1.1: (generalize 2.4.2)

f_n and $\frac{\partial f_n}{\partial x_m}$ cont. for all n,m. in neighborhood of IC

$\Rightarrow$ unique soln exists at least for some time h.



Linear

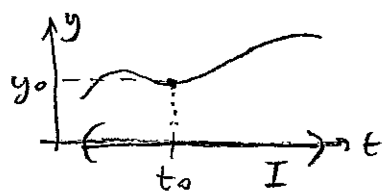
1st-order

standard form $y' + p(t)y = g(t)$, $y(t_0) = y_0$

Thm 2.4.1:

p, g cont. in time interval I

$\Rightarrow$ unique soln everywhere in I.



2nd-order

$y'' + p(t)y' + q(t)y = g(t)$

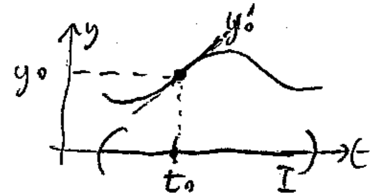
$y(t_0) = y_0$

$y'(t_0) = y_0'$

Thm 3.2.1: (generalizes 2.4.1)

p, q, g cont. in I

$\Rightarrow$ unique soln. everywhere in I



1st-order systems

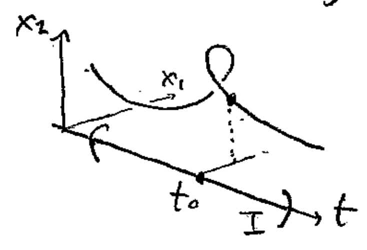
$\begin{cases} x_1' = p_{11}(t)x_1 + \cdots p_{1n}(t)x_n + g_1(t) \\ \vdots \\ x_n' = p_{n1}(t)x_1 + \cdots p_{nn}(t)x_n + g_n(t) \end{cases}$

& ICs

Thm 7.1.2: (generalizes 2.4.1)

p_{nm}, g_n cont. in I for all n,m

$\Rightarrow$ unique soln. everywhere in I



Barnett
10/27/05

note, IC values y_0, y_0' irrelevant for linear cases!