Math 23 Diff Eq: In-class Midterm (2005)

Note this was just over half the time you will be given. Your exam will therefore have more questions.
65 minutes, 65 points. Answer all five questions, giving as much explanation as you have time for. No calculator needed; no algebra-capable ones allowed.

1. [10 points] Find the general solution to $ty' + 2y = 3t - 2$, for $t > 0$. Is the $t \to \infty$ behavior stable or unstable? To what, if any, function of t is this solution asymptotic?

2. [8 points] Find the *general* solution to $y'' + 4y' + 4y = t$

3. [21 points] Solve the following initial-value problems. In each case explain why your solution is the only solution, or find another solution (NB 4 points are reserved for this in each case so put in corresponding detail).

   (a) $y' = ty^{1/2}$ with $y(0) = 0$.

   (b) $y^2 + (2xy + 1)y' = 0$ with $y(0) = 1$. (Remember to explain or find another solution as before...)

4. [10 points]

   (a) Solve the initial-value problem $y'' + y = \cos t$ with $y(0) = 0$ and $y'(0) = 0$. Note this is a driven mass-spring system released from rest.

   (b) What is the domain of t over which your solution is guaranteed to exist? (explain)

5. [16 points] Consider $y'' - x^2 y = 0$.

   (a) Is $x_0 = 0$ a regular point? (explain your answer)

   (b) Find the general power-series solution about $x_0 = 0$ writing the answer in the form $c_1 y_1(x) + c_2 y_2(x)$, where only the first 3 terms each of y_1 and y_2 need be given.

   (c) Demonstrate that the y_1 and y_2 you found form a fundamental set of solutions.

   (d) What is the most you can state about the radius of convergence of the series?