## Math 23 Diff Eq: Quiz 2 (Linear $2^{\text {nd }}$-order ODEs)

25 minutes, 25 points. Answer all questions, giving as much explanation as you have time for. No calculator needed; no algebra-capable ones allowed.

1. [7 points]
(a) Write down a general form of the solution to $y^{\prime \prime}-2 y^{\prime}+5 y=0$.
(b) Compute the Wronskian of $e^{-2 t}$ and $t e^{-2 t}$. If these were solutions of an ODE of the form $y^{\prime \prime}+$ $p(t) y^{\prime}+q(t) y=0$, what must you conclude about $p(t) ?$
2. [10 points] Consider $y^{\prime \prime}+7 y^{\prime}+12 y=e^{-3 t}$.
(a) Write down the corresponding homogeneous general solution.
(b) Use the method of undertermined coefficients to find a particular solution
(c) Solve the ODE given $y(0)=2$ and $y^{\prime}(0)=-7$.
(d) Imagine the right-hand side $\left(g(t)\right.$ term) were changed to $t^{2}$. Write down the trial form you would choose for the particular solution [Bonus if time: solve it!]
3. [8 points] Using variation of parameters, find a particular solution to

$$
y^{\prime \prime}+4 y^{\prime}+4 y=\frac{e^{-2 t}}{t^{2}} \quad \text { for } t>0
$$

Finally, use this to write down the general solution.

