Math 23 Diff Eq: Quiz 4 (Phase plane, BVPs, Fourier, heat eqn)

25 minutes, 25 points. Answer all questions, giving as much explanation as you have time for. No calculator needed; no algebra-capable ones allowed.

1. [5 points] Consider the ODE system

$$\begin{aligned} x' &= -y + xy \\ y' &= \sin x \end{aligned}$$

Linearize the system about the critical pont $(0, 0)$, that is, give the A matrix. [Hint: what is linearization of sin?]

From the stability of the linearized system (being what?), what can you deduce about the stability of the nonlinear system?

2. [7 points] Solve the following boundary-value problems, stating if a solution *exists* and *is unique*. If it exists, sketch it.

   (a) $u'' + u = 0$, $u(0) = 1$, $u(\pi) = 0$.

(b) $u'' + u = 0$, $u'(0) = 0$, $u'(\pi) = 0$.

3. [6 points] Calculate the coefficients in the *Fourier sine series* for the function $f(x) = 1/2$ in the interval $0 < x < \pi$. Try to simplify your answer so that it doesn't involve trig functions.

Write out the first 3 terms of the series here: $f(x) =$

4. [7 points] a) A rod of length π has both ends fixed to temperature zero. The temperature is initially 50 in $0 < x < \pi$. Write the formula for the resulting temperature $u(x,t)$ as a funciton of space and time.

b) Suppose the right end were instead held at temperature 100π, the left end still at zero, and the initial temperature was instead $u(x,0) = 50 + 100x$. Write down the solution [Hint you can reuse the above results]: