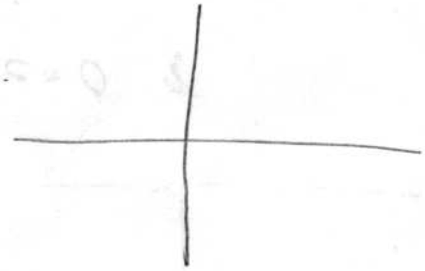
MATH 23 : Complex worksheet.

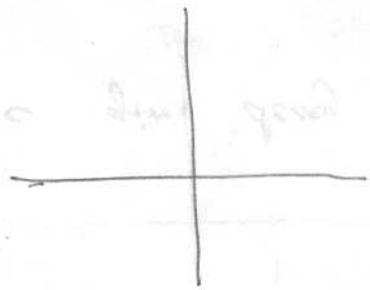
Barnett
10/10/05.
& 10/15/07

Write as $A + iB$ with A, B real, and plot on Argand diagram:

i) $e^{i\pi/4}$



ii) $2e^{i\pi/3}$



iii) $e^{\lambda \pm i\mu}$ with $\lambda = -1$, $\mu = +1$

Sketch curves as t increases from 0 to $+\infty$ of the path in complex plane:

$y(t) = e^{(\lambda + i\mu)t}$

$\lambda < 0 \ , \ \mu > 0$

same but $\lambda > 0 \ , \ \mu > 0$

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~ SOLUTIONS ~

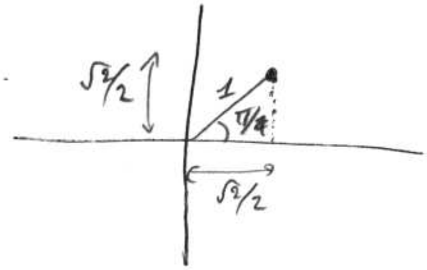
Barnett
10/10/05.
& 10/15/07

Write as $A + iB$ with A, B real, and plot on Argand diagram:

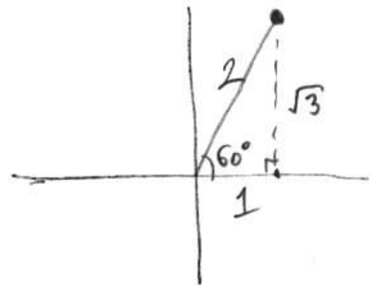
radians → 45°

i) $e^{i\pi/4} = \cos \pi/4 + i \sin \pi/4$

$= \frac{\sqrt{2}}{2} + i\frac{\sqrt{2}}{2}.$



ii) $2e^{i\pi/3} = 1 + i\sqrt{3}$ — 60°



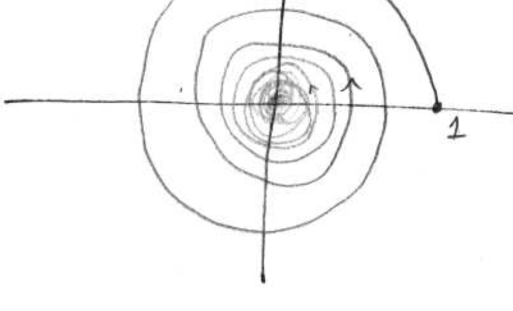
iii) $e^{\lambda \pm i\mu}$ with $\lambda = -1$, $\mu = +1$

$= e^{\lambda} e^{\pm i\mu}$ $= e^{-1} e^{\pm i}$

$= \frac{1}{e}(\cos 1 \pm i \sin 1)$

by rule for exponentials.

Sketch curves as t increases from 0 to $+\infty$ of the path in complex plane:

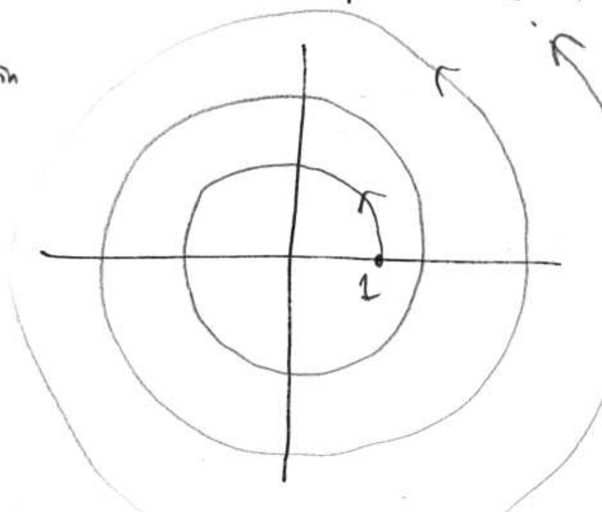


decay to origin since $\lambda < 0$

$e^{\lambda t} \to 0$.

this is the radius at each t.

spiral in



spiral out

$y(t) = e^{(\lambda + i\mu)t}$

$\lambda < 0$, $\mu > 0$

same but $\lambda > 0$, $\mu > 0$.

$e^{\lambda t}$ growing arbitrarily large