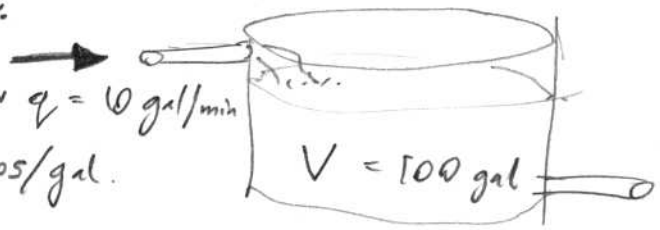
MATH 23 WORKSHEET : Modeling — 10/5/07 Barnett

Continue the tank problem:



$$\frac{dy}{dt} = q\,c_{in} - q\,c_{out}(t)$$

a) Write $c_{out}(t)$ in terms of $y(t)$:

What assumption did you make about behavior of salt in tank?
(Discuss with neighbor!)

b) Write it as a driven 1st order lin. ODE

$$y' + ry = g(t)$$

What is r?
What is $g(t)$?

c) Solve the ODE:

d) When does c_{out} reach 1 lb/gal?
[Hint: $\ln 2 \approx 0.69$]

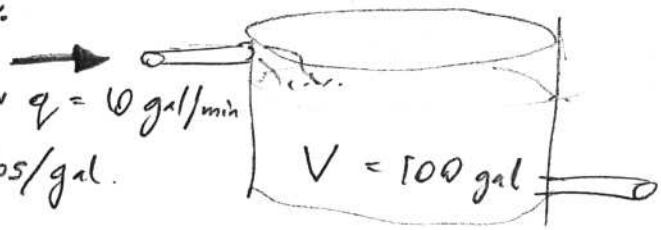
e) What if $c_{in} = c_{in}(t) = 1 + \sin \omega t$, a periodic-in-time pollutant?

MATH 23 WORKSHEET : Modeling — SOLUTIONS

10/5/07
Barnett

Continue the tank problem:

$y(t)$ = lbs of salt in tank



salt input conc. is c_{in} = 2 lbs/gal (given)

ICs $y(0) = 0$.

$c_{out}(t)$ = output concentration, want to know.

$$\frac{dy}{dt} = q\,c_{in} - q\,c_{out}(t)$$

a) Write $c_{out}(t)$ in terms of $y(t)$: $c_{out}(t) = \frac{y(t)}{V}$ (lbs / gals)

units: lb/gal

What assumption did you make about behavior of salt in tank? (Discuss with neighbor!)

salt is perfectly evenly mixed in container.

b) Write it as a driven 1st order lin. ODE

$$y' + ry = g(t)$$

sub. in c_{out}

$$y' + \frac{q}{V}y = qc_{in}$$

What is r? $r = q/V$

What is $g(t)$? qc_{in}, constant function.

c) Solve the ODE:

$$y(t) = \frac{1}{\mu(t)}\left[\int \mu(t)\,g(t)\,dt + c\right] \qquad \mu(t) = e^{rt}$$

$$= e^{-rt}\left[\int e^{rt} q c_{in}\,dt + c\right]$$

$$= \frac{qc_{in}}{r} + ce^{-rt} \qquad (qc_{in}/r = c_{in}V)$$

c must equal $-\frac{qc_{in}}{r}$ to match IC $y(0)=0$.

$$= c_{in}V\left(1 - e^{-rt}\right) \qquad (200.)$$

d) When does c_{out} reach 1 lb/gal?

[Hint: $\ln 2 \approx 0.69$]

$$c_{out}(t) = c_{in}\left(1 - e^{-\frac{q}{V}t}\right) = \frac{1}{2}c_{in} \text{ when } e^{-\frac{q}{V}t} = \frac{1}{2} \text{ so } t = \frac{V}{q}\ln 2 \approx 6.9 \text{ min}$$

e) What if $c_{in} = c_{in}(t) = 1 + \sin \omega t$, a periodic-in-time pollutant?

see lecture 6.