

# Some useful Taylor series

$$\frac{1}{1-x} = 1 + x + x^2 + x^3 + \dots = \sum_{n=0}^{\infty} x^n \quad \text{for } -1 < x < 1 \quad (\text{so } R = 1)$$

$$e^x = 1 + x + \frac{x^2}{2} + \frac{x^3}{6} + \dots = \sum_{n=0}^{\infty} \frac{x^n}{n!} \quad R = \infty$$

$$\cos x = 1 - \frac{x^2}{2} + \frac{x^4}{4!} + \dots = \sum_{n=0}^{\infty} \frac{(-1)^n x^{2n}}{(2n)!} \quad R = \infty$$

$$\sin x = x - \frac{x^3}{3!} + \frac{x^5}{5!} + \dots = \sum_{n=0}^{\infty} \frac{(-1)^n x^{2n+1}}{(2n+1)!} \quad R = \infty$$

$$\frac{1}{x} = \frac{1}{1-(1-x)} = 1 + (1-x) + (1-x)^2 + \dots = \sum_{n=0}^{\infty} (1-x)^n \quad \text{for } 0 < x < 2 \quad (\text{so } R = 1)$$

$$\ln x = \int \frac{1}{x} dx = C + x - \frac{(1-x)^2}{2} - \frac{(1-x)^3}{3} - \dots = \quad \text{for } 0 < x < 2 \quad (\text{so } R = 1)$$

(plug in  $x = 1$  to find  $C = -1$ )  $= \sum_{n=0}^{\infty} (-1)^{n+1} (x-1)^n$

$$\tan^{-1} x = \int \frac{1}{1+x^2} dx = \sum_{n=0}^{\infty} \frac{(-1)^n x^{2n+1}}{2n+1} \quad \text{for } -1 < x < 1 \quad (\text{so } R = 1)$$

(same strategy as for  $\ln x$ )