

Math 23 Fall 2014
Differential Equations

Final Exam

Friday, November 21, 3:00PM - 6:00PM

Your name (please print): _____

Section (circle one): Olivia Prosper, Min Hyung Cho

Instructions: This is a closed book, closed notes exam. The use of calculators is not permitted. The exam consists of **9** problems and this booklet contains **15** pages (including this one). **On problems 2 through 9, you must show your work and justify your assertions to receive full credit.** Justify your answers and simplify your results as much as possible. Also, please clearly mark your final (simplified) answer. The last two pages of this booklet are blank. Good Luck!

The Honor Principle requires that you neither give nor receive any aid on this exam.

Math 23 Fall 2014

Your name (please print): _____

Problem	Points	Score
1	10	
2	6	
3	10	
4	12	
5	10	
6	20	
7	11	
8	10	
9	11	
Total	100	

Work will not be graded in this page

1. Short Answer Questions ((a) ~ (b))-Work will not be graded

- (a) (**5 points**) Consider a uniform rod of length π for which $\alpha^2 = 1$. Then, the temperature of the rod follows

$$u_t = u_{xx}, \quad 0 < x < \pi.$$

Suppose the initial temperature is given by $u(x, 0) = \sin x + \cos x$ and both ends of the rod are insulated. Find the steady state temperature in the bar.

Answer: _____

- (b) (**5 points**) Determine the lower bound for the radius of convergence of the power series solution near $x_0 = 1$ of the differential equation

$$(x^2 + 4)(x + 2)y'' + (x - 2)y' - x^2y = 0.$$

Answer: $\rho =$ _____

Show your work

2. **(6 points)** Determine whether the following set of vectors is linearly independent or linearly dependent:

$$\vec{v}_1 = \begin{pmatrix} 1 \\ 3 \\ -1 \\ 0 \end{pmatrix}, \vec{v}_2 = \begin{pmatrix} 0 \\ 1 \\ 0 \\ 1 \end{pmatrix}, \vec{v}_3 = \begin{pmatrix} 1 \\ 5 \\ -1 \\ 2 \end{pmatrix}.$$

Answer: _____

Show your work

3. (10 points) Find an explicit solution of the initial value problem.

$$\frac{dP}{dt} = (1 - P)P, P(0) = \frac{1}{2}$$

Answer: $P(t) =$ _____

Show your work

4. (12 points) Determine if $f(x) = e^{-x}$ and $g(x) = -x + 1$ are orthogonal on $[0, \infty)$ and justify.

Answer: _____

Show your work

5. (10 points) Seek a power series solution (find a recurrence relation) of

$$y^{(3)} + y' = 0$$

near $x_0 = 0$ by assuming a_0 , a_1 , and a_2 are arbitrary constants.

Answer: $a_{n+3} =$ _____

Show your work

6. An engineer attempts to take damping into account in the vibrating string problem by using the following partial differential equation and boundary conditions.

$$\begin{cases} u_{tt} = u_{xx} - 2u_t & 0 < x < \pi, t > 0 \\ u(0, t) = u(\pi, t) = 0 & t \geq 0 \\ u_t(x, 0) = \sin 2x & 0 \leq x \leq \pi \\ u(x, 0) = 0 & 0 \leq x \leq \pi \end{cases}$$

- (a) **(10 points)** Use separation of variables $u(x, t) = X(x)T(t)$ to find a set of two second order differential equations satisfied by X and T . Please specify the boundary condition for X .

Answer: Equation for X : _____

Answer: Boundary conditions for X : _____

Answer: Equation for T : _____

Part (b) next page →

Show your work

(b) (10 points) Find the displacement $u(x, t)$

Answer: $u(x, t) =$ _____

Show your work

7. (a) (2 points) Draw the graph of

$$f(x) = \begin{cases} 0, & -\pi < x \leq -\pi/2 \\ -2, & -\pi/2 < x \leq 0 \\ 2, & 0 < x \leq \pi/2 \\ 0, & \pi/2 < x \leq \pi \end{cases}, \quad f(x + 2\pi) = f(x).$$

Part (b) next page →

Show your work

(b) (9 points) Find the Fourier series of $f(x)$.

Answer: $f(x) =$ _____

Show your work

8. (10 points) Either solve the boundary value problem, or show that it has no solution.

$$y'' + 5y = \cos(x), \quad y'(0) = 0, \quad y\left(\frac{\pi}{\sqrt{5}}\right) = 0.$$

Answer: _____

Show your work

9. (11 points) Solve the differential equation $\vec{x}' = A\vec{x}$, where $A = \begin{pmatrix} 5 & -3 \\ 3 & -1 \end{pmatrix}$.

Answer: $\vec{x} =$ _____

Extra page for scratch work. I will not grade work on this page unless you write on another page "problem continued on page 14".

Extra page for scratch work. I will not grade work on this page unless you write on another page “problem continued on page 15”.