Math 23, Practice set for the midterm exam

1. The general solution of the differential equation $y' + 2y = 3t^2 e^{-2t}$ is:

   (a) $3t^2 e^{-2t} + Ce^{-2t}$
   (b) $t^3 e^{-2t} + C$
   (c) $t^3 e^{-2t} + Ce^{-2t}$
   (d) $t^3 e^{2t} + Ce^{2t}$

2. The solution to the initial value problem $y'' + 2y' + 2y = 5e^x,\ y(0) = 1,\ y'(0) = 2$, is:

   (a) $y(x) = e^{-x}\cos x + e^{-x}\sin x + e^x$
   (b) $y(x) = e^x \cos x + e^x \sin x + 5e^x$
   (c) $y(x) = e^{-x}\sin x + e^x$
   (d) $y(x) = e^{-x}(\cos x + 3\sin x)$

3. Which of the following forms a set of fundamental solutions for the equation

$$y'' - 4y' + 4y = 0.$$

   (a) $y_1(x) = e^{2x},\ y_2(x) = e^{-2x}$
   (b) $y_1(x) = e^{2x},\ y_2(x) = (x+1)e^{2x}$
   (c) $y_1(x) = e^{-2x},\ y_2(x) = 2xe^{-2x}$
   (d) $y_1(x) = e^{-2x},\ y_2(x) = (x+1)e^{-2x}$

4. Which of the following is the correct form of a particular solution for the equation

$$y'' - 5y' + 6y = \cos 3x + e^{2x}\ ?$$

   (a) $Ae^{2x} + B\cos 3x + C\sin 3x$
   (b) $Ae^{2x} + Bx\cos 3x + Cx\sin 3x$
   (c) $Axe^{2x} + Bx\cos 3x + Cx\sin 3x$
   (d) $Axe^{2x} + B\cos 3x + C\sin 3x$

5. Given that $y_1(t) = t$ is a solution of $t^2y'' + ty' - y = 0,\ t > 0$, find its general solution.

6. A differential equation has the form $y'' + p(t)y' + q(t)y = 0$, where p and q are both continuous on the interval $-1 < t < 1$. Explain why it is not possible that the general solution to that equation on that interval is $y = At^2 + Bt$.

7. Consider the differential equation

$$\overbrace{t^2y'' - t(t+2)y' + (t+2)y}^{L[y]} = 2t^3 \qquad (t > 0).$$

The functions $y_1(t) = t$ and $y_2(t) = te^t$ are both solutions to the corresponding homogeneous equation $L[y] = 0$. Use the method of variation of parameters to find a particular solution to the original equation.

8. A boater and a motor boat together weigh 640 lb. The motor provides a constant driving force of 200 lb in the direction of motion, and the resistance of the water is equal numerically to twice the velocity (in feet per second). If the boat is initially at rest, find the time when the velocity equals to 50 ft/sec. ($g = 32\text{ft/sec}^2$)

9. A box weighing 6.4 pounds is suspended from a spring hanging inside a fluid-filled cylider. (This means that mg is equal to 6.4 pounds, where m is the mass of the box and g is the acceleration of gravity, $32\frac{ft}{sec^2}$.) The spring constant is $k = .6\frac{lb}{ft}$. The damping constant, for the damping induced by the liquid in the cylinder, is $.4\frac{lb\,sec}{ft}$. The box is hanging at rest, in equilibrium. Then (say at time $t = 0$) the box is given a sharp tap that sends it downward at a speed of 2 feet per second. The box is observed to bob up and down with smaller and smaller bobs, approaching its rest position.

   (a) Find the displacement of the box from its rest position at any time $t > 0$.

   (b) Suppose you want to fill the cylinder with a more viscous liquid, so that the box will return to its rest position without bobbing up and down and up and down. If you are using the same spring, what should your damping coefficient be?

10. What is the solution of the linear system $\begin{pmatrix} x \\ y \end{pmatrix}' = \begin{pmatrix} 4 & 5 \\ -4 & -4 \end{pmatrix} \begin{pmatrix} x \\ y \end{pmatrix}$ if $x(0) = 1$ and $y(0) = 2$?

11. Consider the linear system: $\begin{pmatrix} x \\ y \end{pmatrix}' = \begin{pmatrix} -3 & 5/2 \\ -5/2 & 2 \end{pmatrix} \begin{pmatrix} x \\ y \end{pmatrix}$.

   (a) Classify the critical point $(0, 0)$ as to type and determine its stability.

   (b) Find the general solution of the linear system.

Answers: 1.C, 2.C, 3.B, 4.D.