MATH 23 FINAL EXAM REVIEW

Try and take this like a real exam. Give yourself two hours or so. No calculators, but you may use the table on page 300.

1. Solve the initial value problem
$$y'' + y = 3\sin 2t - (3\sin 2t)u_{2\pi},\ y(0) = 1,\ y'(0) = -2.$$
2. Find the general solution of the system
$$\mathbf{x}' = \begin{pmatrix} 1 & 1 \\ 4 & -2 \end{pmatrix} \mathbf{x}$$
and plot a few trajectories of the system. Be sure to indicate behavior as $t \to \pm\infty$
3. Find the general solution of the system
$$\mathbf{x}' = \begin{pmatrix} 1 & 0 & 0 \\ 2 & 1 & -2 \\ 3 & 2 & 1 \end{pmatrix} \mathbf{x}$$
4. Find a solution to the initial value problem
$$2u_{xx} = u_t,\ u(0,t) = u(\pi,t) = 0,\ u(x,0) = \sin 3x - \sin 5x$$
5. Consider the function
$$f(x) = \begin{cases} 0, 0 \leq x \leq 1, 2 \leq x \leq 3 \\ 1, 1 \leq x \leq 2 \end{cases}$$
   (a) Graph the even and odd periodic extensions of this function, with period 6.
   (b) Find the Fourier sine and cosine series for the appropriate functions in part (a).
   (c) Draw the functions to which the series in part (b) converge to for three periods.
6. Consider the differential equation
$$u_{xx} + u_{xt} + u_t = 0$$
   (a) Using separation of variables, replace the PDE with a pair of ODE's
   (b) Solve these ODE's to give a general solution for $u(x,t)$. You may consider only the case where the ratio of X dependent terms is positive.
7. (a) Prove that the Fourier cosine Series for x^2, with period 2π is
$$\frac{\pi^2}{3} + 4\sum_{n=1}^{\infty} \frac{(-1)^n}{n^2} \cos nx$$

(b) Use this result to show that

$$\frac{\pi^2}{6} = \sum_{n=1}^{\infty} \frac{1}{n^2}.$$

8. Write an essay on how you will spend your spring break.