

# Math 24 Spring 2006 Midterm Review Guide

Some items to know:

- (1) Definition of vector (sub)space, how to tell whether a given set and operations form a vector (sub)space
- (2) Result of union or intersection of subspaces
- (3) Definition of linear combination, span, generate, linear dependence, linear independence, relationships among these definitions
- (4) What linear dependence and independence of one set  $S_1 \subset S_2$  tells you about the other set (if anything)
- (5) Definition of (ordered) basis; relationship of basis size to size of linearly independent sets and spanning sets, number of representations of a vector of  $V$  as linear combinations of its basis vectors
- (6) The standard (ordered) bases for the key examples of vector spaces
- (7) How to obtain a basis from a spanning set or linearly independent set (in particular, that it can always be done)
- (8) Definition of dimension, dimension of subspace
- (9) Definition of linear transformation, null space/kernel, range/image (special examples of the identity transformation and zero transformation)
- (10) How to tell whether a map  $V \rightarrow W$  is linear
- (11) That  $N(T)$  and  $R(T)$  are subspaces of  $V$  and  $W$ , respectively, and the relationship between nullity( $T$ ), rank( $T$ ), and  $\dim(V)$  (and that the relationship is only valid for finite-dimensional  $V$ )
- (12) How to find a spanning set for  $R(T)$
- (13) The relationship between nullity( $T$ ) and whether  $T$  is 1-1 (and, if  $V, W$  have equal finite dimension, between those and  $T$  being onto)
- (14) That linear transformations are uniquely determined by what they do to bases of  $V$ , and that anything one wants to map the basis elements to gives a linear transformation
- (15) How to represent vectors as coordinate vectors and linear transformations as matrices, and use that to find the image of the vector
- (16) That the collection of all linear transformations from  $V$  to  $W$ ,  $\mathcal{L}(V, W)$ , is a vector space whose addition and scalar multiplication correspond neatly to the addition and scalar multiplication of matrices representing linear transformations
- (17) How to perform matrix multiplication, and the connection between that and composition of linear transformations (and that both are well-behaved although not commutative)

Do not worry about  $L_A$ , which we began at the very end of Friday's class (p. 92-95 of section 2.3 will not be on the midterm). Skip groups and fields. If you need sum, direct sum, Kronecker delta,  $T$ -invariance and restriction, projection on  $W_1$  along  $W_2$ , or other particular linear transformations which have names in the book but are not mentioned in the list above, they will be given to you.