cyclotomic

```
(x^(12)-1).factor()
```

$$\left(x^4 - x^2 + 1\right)\left(x^2 + x + 1\right)\left(x^2 - x + 1\right)\left(x^2 + 1\right)(x + 1)(x - 1)$$

```
range?
```

Type: <type 'builtin_function_or_method'>

Definition: range([noargspec])

Docstring:

```
range(stop) -> list of integers
range(start, stop[, step]) -> list of integers

Return a list containing an arithmetic progression of integers.
range(i, j) returns [i, i+1, i+2, ..., j-1]; start (!) defaults to 0.
When step is given, it specifies the increment (or decrement).
For example, range(4) returns [0, 1, 2, 3].  The end point is omitted!
These are exactly the valid indices for a list of 4 elements.
```

```
range(1,13)
```

$$[1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12]$$

```
for n in range(1,13):
    n,(x^n -1).factor()
```

$$(1, x - 1)$$

$$(2, (x + 1)(x - 1))$$

$$\left(3, \left(x^2 + x + 1\right)(x - 1)\right)$$

$$\left(4, \left(x^2 + 1\right)(x + 1)(x - 1)\right)$$

$$\left(5, \left(x^4 + x^3 + x^2 + x + 1\right)(x - 1)\right)$$

$$\left(6, \left(x^2 + x + 1\right)\left(x^2 - x + 1\right)(x + 1)(x - 1)\right)$$

$$\left(7, \left(x^6 + x^5 + x^4 + x^3 + x^2 + x + 1\right)(x - 1)\right)$$

$$\left(8, \left(x^4 + 1\right)\left(x^2 + 1\right)(x + 1)(x - 1)\right)$$

$$\left(9, \left(x^6 + x^3 + 1\right)\left(x^2 + x + 1\right)(x - 1)\right)$$

$$\left(10, \left(x^4 + x^3 + x^2 + x + 1\right)\left(x^4 - x^3 + x^2 - x + 1\right)(x + 1)(x - 1)\right)$$

$$\left(11, \left(x^{10} + x^9 + x^8 + x^7 + x^6 + x^5 + x^4 + x^3 + x^2 + x + 1\right)(x - 1)\right)$$

$$\left(12, \left(x^4 - x^2 + 1\right)\left(x^2 + x + 1\right)\left(x^2 - x + 1\right)\left(x^2 + 1\right)(x + 1)(x - 1)\right)$$

```
cyclotomic_polynomial(11)
```

$$x^{10} + x^9 + x^8 + x^7 + x^6 + x^5 + x^4 + x^3 + x^2 + x + 1$$

```
from sage.rings.polynomial.cyclotomic import cyclotomic_coeffs
```

```
cyclotomic_coeffs(12)
```

$$\{0 : 1, 2 : -1, 4 : 1\}$$

```
cyclotomic_coeffs(12, sparse=false)
```

$$[1, 0, -1, 0, 1]$$

```
for n in range(10,20):
    n,cyclotomic_coeffs(n,sparse=false)
```

$$(10, [1, -1, 1, -1, 1])$$

$$(11, [1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1])$$

$$(12, [1, 0, -1, 0, 1])$$

$$(13, [1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1])$$

$$(14, [1, -1, 1, -1, 1, -1, 1])$$

$$(15, [1, -1, 0, 1, -1, 1, 0, -1, 1])$$

$$(16, [1, 0, 0, 0, 0, 0, 0, 0, 1])$$

$$(17, [1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1])$$

$$(18, [1, 0, 0, -1, 0, 0, 1])$$

$$(19, [1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1])$$

Max and min of coefficients of cyclotomic polynomials

```
for n in range(1,100):
    list= cyclotomic_coeffs(n, sparse=false)
```

```
n,min(list),max(list)
```

```
(1, -1, 1)
(2, 1, 1)
(3, 1, 1)
(4, 0, 1)
(5, 1, 1)
(6, -1, 1)
(7, 1, 1)
(8, 0, 1)
(9, 0, 1)
(10, -1, 1)
(11, 1, 1)
(12, -1, 1)
(13, 1, 1)
(14, -1, 1)
(15, -1, 1)
(16, 0, 1)
(17, 1, 1)
(18, -1, 1)
(19, 1, 1)
(20, -1, 1)
(21, -1, 1)
(22, -1, 1)
(23, 1, 1)
(24, -1, 1)
(25, 0, 1)
(26, -1, 1)
(27, 0, 1)
(28, -1, 1)
(29, 1, 1)
(30, -1, 1)
(31, 1, 1)
(32, 0, 1)
(33, -1, 1)
(34, -1, 1)
(35, -1, 1)
(36, -1, 1)
(37, 1, 1)
(38, -1, 1)
(39, -1, 1)
(40, -1, 1)
(41, 1, 1)
(42, -1, 1)
(43, 1, 1)
(44, -1, 1)
(45, -1, 1)
(46, -1, 1)
(47, 1, 1)
```

```
(48, -1, 1)
(49, 0, 1)
(50, -1, 1)
(51, -1, 1)
(52, -1, 1)
(53, 1, 1)
(54, -1, 1)
(55, -1, 1)
(56, -1, 1)
(57, -1, 1)
(58, -1, 1)
(59, 1, 1)
(60, -1, 1)
(61, 1, 1)
(62, -1, 1)
(63, -1, 1)
(64, 0, 1)
(65, -1, 1)
(66, -1, 1)
(67, 1, 1)
(68, -1, 1)
(69, -1, 1)
(70, -1, 1)
(71, 1, 1)
(72, -1, 1)
(73, 1, 1)
(74, -1, 1)
(75, -1, 1)
(76, -1, 1)
(77, -1, 1)
(78, -1, 1)
(79, 1, 1)
(80, -1, 1)
(81, 0, 1)
(82, -1, 1)
(83, 1, 1)
(84, -1, 1)
(85, -1, 1)
(86, -1, 1)
(87, -1, 1)
(88, -1, 1)
(89, 1, 1)
(90, -1, 1)
(91, -1, 1)
(92, -1, 1)
(93, -1, 1)
(94, -1, 1)
(95, -1, 1)
```

```
(96, -1, 1)
(97, 1, 1)
(98, -1, 1)
(99, -1, 1)
```