WRITTEN HW #2, DUE ...

(1) (10 points) Suppose that we perform the Euclidean algorithm on a pair of positive integers a, b, where $a > b$, and obtain a sequence of remainders $r_0 = b, r_1, r_2, \ldots, r_n = 0$. Show that $r_{i+2} < r_i/2$ for all $0 \leq i \leq n-2$.

(2) (10 points) Suppose that we perform the Euclidean algorithm on a pair of positive integers a, b, where $a > b$. Show that the algorithm terminates after at most $\lfloor 2\log_2 a + 3 \rfloor$ Euclidean divisions. Hint: Use the previous problem! ($\lfloor x \rfloor$ means the largest integer less than or equal to x, and is the same thing as rounding a real number down.)

(3) (5 points each)
  (a) Find all integer solutions (x, y) to $77x + 98y = 7$.
  (b) Find all integer solutions (x, y) to $6x + 21y = 9$.

(4) (10 points) The residents of a parallel universe play a simplified version of (American) football where teams can only score on touchdowns and field goals, which are worth 7 and 3 points respectively. What is the largest number of points which cannot be achieved by any combination of scores? (Remember to prove your answer!)

(5) (10 points) Let $a, b > 0$ be two positive integers. Consider the line segment connecting $(0, 0)$ to (a, b). A *lattice point* is a point (x, y) where both x, y are integers. Show that this line segment passes through exactly $\gcd(a, b) + 1$ lattice points. (We count both $(0, 0)$ and (a, b) as lying on the segment.)