WRITTEN HW #7, DUE NOV 16 2011

Remember to write clearly and to justify all your claims in your solutions. Please staple your assignment before turning it in.

(1) (10 points) Find all solutions to $x^2 + 3x + 3 \equiv 0 \bmod 7^3$. All calculations should be done by hand.
(2) (10 points) Show that $x^3 \equiv 9 \bmod 11^n$ always has a solution if $n \geq 1$.
(3) (10 points) Suppose G is a cyclic group of order n with generator g. Recall that the order of every element in G divides n. Suppose $d \mid n, d \geq 1$. How many elements of G have order d? What are they, in terms of g?
(4) (10 points) Suppose $g_1 \in G_1$ has order n_1 and $g_2 \in G_2$ has order n_2. What is the order of $(g_1, g_2) \in G_1 \times G_2$, in terms of n_1, n_2?
(5) (10 points) Show that each of the following groups is isomorphic to $\mathbb{Z}/n\mathbb{Z}$:
   - The nth roots of unity; ie, the complex roots of $x^n = 1$, under multiplication.
   - The rotational symmetries of the regular n-gon, under composition.